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A dynamic programming simulation of optimal monetary policies designed to stabilize prices and employment

by

Charles Milton Sivesind

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

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For the Graduate College

Iowa State University Ames, Iowa

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CHAPTER I

Section A

"For eight years economic policy and the news about the economy have been dominated by inflation. The story has been a frustrating one. Over the period 1965 to the end of 1973 consumer prices rose by 45 per cent, or at an average rate of 4.8 per cent a year... Many programs have been launched to stop it without durable success. Inflation seemed a Hydraheaded monster, growing two new heads each time one was cut off." (Economic Report of the President, 1974, p. 21).

The tone of the opening paragraph of the 1974 Economic Report of the President is indicative of the difficulties encountered by economic analysts in dealing with the problem of price level determination. Indeed the President's opening statement only revealed the barest outline of the problems actually being faced. At the time the report was issued the U.S. was experiencing the worst sustained peacetime inflation in its history. Consumer prices rose at a projected annual rate of nearly 13% during the first quarter of 1974 as compared to 4% in 1965. At the same time the economy which had been sluggish for some time appeared to be heading for a recession - an absolute decline in real output was predicted.

Standard theories of price determination are incapable of accounting for the simultaneous presence of both these phenomena.

Money matters. Most professional economists and virtually all laymen would agree. The striking constancy of the ratio of the money stock to the level of nominal income has been repeatedly demonstrated. However there is surprisingly little agreement on the mechanism by which changes in nominal income are divided between changes in prices and real output. This "Division Problem" in its various forms is perhaps the single most important unsolved question in macro-economic theory. The failure of macro economists to systematically attack the division problem lies at the root of the inability of policy makers to understand the inflation mechanism and hence to conduct a rational monetary policy.

There are three aspects to the present study. It is first necessary to develop theoretical models within which the Division Problem may be approached. Next it is helpful to obtain some qualitative and quantitative appreciation of the relationships being dealt with. The behavior of the system under various monetary control strategies will be investigated. Finally the results of empirical estimation of several of the critical relationships will be reported. (There are an overwhelming number of plausible causal relationships among the variables which were investigated. Many of these hypotheses have become part of the conventional wisdom without any econometric foundations. Contrary to the

usual practice in the profession, the author feels that reporting of glaring nonrelationships in such cases constitutes as valuable an addition to the body of knowledge as the reporting of regressions which resulted in significant t statistics.)

Students of the art of economic analysis have not reached a consensus on the nature of the solution to the division problem. This is not to say that the problem has received no attention, at least in its various aspects. Indeed, the literature is voluminous. A brief survey of the principal theoretical and empirical approaches occupies the remainder of this chapter. This is not intended to serve as an exhaustive history of thought, but rather as a guide to the general approaches taken and to some extent as a foil for the succeeding analysis. The survey of empirical work related to the division problem will be limited to studies of inflation. The inadequacy of current inflation theory in a sense is the principal weak link in macro economic analysis.

In economic theory the determination of real output and the determination of the average level of prices have typically been approached as separate problems. In the Classical system real output is determined in the real sector alone; quantities of all commodities supplied and demanded in the market place are independent of the price level. Nominal prices are then determined by the Quantity Equation, Mv = Py.

An increase in the money stock creates excess demands in the various commodity and factor markets driving up the average level of prices to a new equilibrium. It is generally maintained that the resulting equilibrium is identical in real terms, and in relative prices, to the initial one. The system is essentially a long run construct. Its defining characteristic is the neutrality of money in long run equilibrium.

The Keynesian system contains no rigorous theory of prices. Real output is determined in the commodity sector as a function of exogenous demand and the rate of interest. In the monetary sector real output, prices, and liquidity preference serve to determine the rate of interest. Curiously, prices are determined outside the system. Keynesian school economists maintain that levels of nominal aggregate demand in excess of the full employment lead to price increases. This system is short run in nature. A principal defining characteristic here is the thesis that monetary disturbances cause corresponding disturbances in the real sector.

In neither of these systems are prices and real output jointly determined endogenous variables. Either the level of real output or the price level is brought in from the outside to close the system. Hence with either system it is theoretically impossible to gain any insight into the mechanism of simultaneous price and output determination.

Construction of a simple macro model in which the level of prices is determined internally will be the first order of business. Milton Friedman (1970) has proposed the basic outline of such a model. Following this lead a simple form of such a model will be developed and its principal characteristics discussed in Chapter II. In Chapter III the principles of dynamic optimization will be applied to specify the conditions under which an optimal monetary policy may be formulated. Chapter IV contains the results of such an optimization technique applied to the model under various sets of initial conditions. Gradient steepest descent techniques are employed to discover numerical solutions to the various problems posed. Finally Chapter V contains some empirical observations on the Phillips Curve approach to price level determination and some empirical support for the price level equation employed in the final model of Chapter II.

Section B

The Quantity Theory is perhaps the most tested proposition in economics. The positive correlation between long term monetary and price movements is well documented (c.f., Friedman, 1956b). However short run price flucuations are not adequately explained by corresponding monetary movements. Economists have been forced to resort to other explanatory variables.

Since the late 1940's economists have taken two main types of empirical approaches to short run price level determination. The oldest and most prolific of these might be called the Phillips tradition. Attention is focused on the apparent tradeoff between employment and inflation. There is an implicit recognition that the level of real output (employment) and the behavior of prices are somehow jointly determined. The precise nature of the linkage being sought is not well specified, unfortunately.

In recent years on the other hand cost determined theories of prices, the so called New Inflation theories, have become popular, primarily one suspects for pragmatic rather than theoretical reasons. Most of the large scale econometric models of the U.S. economy make use of this approach. The linkages in these models between price and real output determination are somewhat weak and indirect. However it will be shown later that even such indirect linkages may give rise to models with desirable long run properties.

Phillips curves have received a vast amount of attention in the published literature. Researchers have delighted in running the Phillips regression under all conceivable variations of functional form using a large variety of variables. Typically the lack of imagination embodied in this approach is matched only by the quantity of obiter dicta generated in

the articles. The net result however has been a general confusion among economists about just what a Phillips curve is and hence whether "it" indeed exists. For present purposes a brief review of the two classic pieces of the genre will serve to demonstrate both the strengths and the ultimate theoretical sterility of this approach.

Economists have been aware of the apparent statistical relationship between unemployment and the rate of change of prices for many years. For example Irving Fisher analyzed the relationship in a paper published in 1926 (see also A. J. Brown, 1955, and B. Hansen, 1951). The Phillips construct was not new. However only recently has the relationship received widespread attention.

Modern interest in the apparent tradeoff between unemployment and wage inflation stems from a 1958 article by A. W. Phillips (Phillips, 1958). Using annual data for nearly 100 years, Phillips discovered a remarkably stable inverse relationship between the level of unemployment and the rate of wage inflation. Further, deviations from this relationship could be explained by reference to the rate of change of unemployment and to "cost of living" factors, particularly a general price index and an index of import prices.

In his opening sentence Professor Phillips set the tone for this and indeed for most subsequent discussions of what

became known as the "Phillips Curve" relationship. "When demand for a commodity or service is high relative to the supply of it we expect the price to rise, the rate of rise being greater the greater the excess demand." (Phillips, 1958, p. 283). This suggests a partial adjustment dynamic process underlying the price determination mechanism. This concept becomes the core feature of the price adjustment sector of the model developed in Chapter II. If the unemployment rate serves as an adequate proxy for the excess supply of labor, two of Phillips' three propositions follow: (1) the rate of wage inflation is inversely related to the level of unemployment, and (2) changes in the rate of unemployment should lead to changes in the rate of wage inflation. Indeed should wage adjustment be somewhat delayed changes in the unemployment rate should precede changes in the rate of wage inflation.

As a final proposition, Phillips suggested that the rate of change of prices, particularly import prices, acts as a third causal influence on wages. In his discussion Phillips refers to these prices as "costs". One gets the distinct feeling that some modified "subsistence wage" theory underlies his discussion: One determinant of the money wage is the cost of maintaining the labor force.

In many ways Phillips' construct is most satisfactory; as compared to many later discussions of the subject it is logical, self contained, and watertight. His article became

immensely popular. The scope of the research is impressive: The relationship holds for over 100 years through a period of rapid industrialization, a major war, and the Great Depression.

The annual rate of wage change series was constructed using first central differences: The rate of change at time t defined as the wage index at t+1 less the index at t-1 divided by twice the index at t. This method results in considerable smoothing of the wage inflation series. Smoothed annual data for the period 1861 to 1913 was blocked by unemployment rate into six groups. Wage rate inflation and unemployment averages were taken within each group. These averages were then fitted to the logarithmic equation

$$\ln(w'/w + a) = \ln(b) + c\ln(u)$$
 (1.1)

where w'/w is the rate of wage change and u is the unemployment rate. Parameters b and c were obtained by least squares estimation; the coding factor a, necessary since w'/w is sometimes negative, was chosen by trial and error. The estimated relationship was

$$w'/w = -0.9 + 9.638u^{-1.394}$$
 (1.2)

Phillips then tested his curve against subsequent data. The remarkable robustness of his hypothesis may be seen in his plot of 1913 to 1948 data (Phillips, 1958, p. 294).

Included in the period are a major wartime inflation and a post-war contraction followed by a monetary disruption inspired by central bank attempts to reestablish the gold standard at pre-war parity with sterling. Finally the period includes the Great Depression of the 1930's.

As examples of his second proposition he identified several subperiods which displayed the counter-clockwise progression of observations which have become known as "Lipsey Loops". In these periods falling unemployment rates were associated with gradually increasing rates of wage inflation and a plot of points above the fitted curve. Following the business cycle peak unemployment increased but wage inflation declined resulting in a plot of points below the line.

Several observations over the data period clearly could not be explained by the hypothesis. In each case these were accompanied by sharp changes in import prices. Hence proposition three.

The Phillips relationship holds up broadly under a wide range of circumstances. Two items however are of special interest. First this original discussion attempted to explain the rate of wage inflation, not increases in the general price level. There is no hint here of wage push inflation; indeed increases in prices lead to increases in wages in this construct. Second the theoretical underpinnings for this discussion hinge on the acceptance of the inverse of the

unemployment rate as an acceptable proxy for the excess demand for labor.

In Phillips' original study only the first of his three propositions was quantified by regression techniques. In a 1960 study Richard Lipsey attempted to close this omission. He began with a similar though modified form of Phillips' equation

$$w'/w = a + b(1/U) + c(1/U)^2$$
 (1.4)

adding in turn the rate of change of unemployment and the rate of change of prices, both expressed in first central differences as in the Phillips study. Phillips' wage series was modified slightly also.

On 1862 to 1913 data he obtained the relationship

$$w'/w = -1.42 + 7.06 (1/U) + 2.31(1/U)^2$$
 (1.5)

The graph of this relationship is very close to that of the original Phillips equation.

Adding the rate of change of U gave

$$w'/w = -1.52 + 7.60(1/U) + 1.61(1/U)^2 - 0.23(U'/U)$$
 (1.6)

Phillips' suggestion that the rate of wage inflation would be unaffected by any rate of price inflation less than the expected rate of wage inflation was rejected by Lipsey. He could find no justification for such an asymmetrical structure. Estimation of the relationship

$$w'/w = a + b(1/U) + c(1/U)^{2} + d(U'/U) + e(P'/P)$$
 (1.7)

gave

$$w'/w = -1.21 + 6.45(1/U) + 2.26(1/U)^{2}$$

-0.019(U'/U) + 0.21(P'/P) (1.8)

suggesting a significant positive relationship between the rate of price inflation and the rate of wage inflation. Reestimating a modification of this equation on post World War I data, 1923 to 1939 and 1948 to 1957, gave

$$w'/w = 0.47 + 0.43(1/U) + 11.18(1/U)^4$$

(2.10) (6.00)

+
$$0.038(U'/U)$$
 + $0.69(P'/P)$ (1.9)
(0.012) (0.08)

(Note: The practice of placing standard errors in parentheses beneath the associated parameter estimates will be followed whenever possible. Standard errors were not always reported in early econometric work, unfortunately.) The sign of the U'/U coefficient has changed; it would appear that the loops have changed direction, from a counter-clockwise to a clockwise progression. Since the size of the P'/P coefficient increased rather sharply, Lipsey concluded that there seemed to be even more rapid adjustment in wages to price increases in the post World War I period than prior to World War I. Indeed by far the most important explanatory variable in Equation 1.9 is the price inflation variable. Wages and prices are highly correlated; hence price movements tend to dominate the relationship.

Lipsey's theoretical model was perhaps the most innovative aspect of the study. His model seems to have been the first explicit attempt to provide some explanation of the microeconomic foundations underlying the observed macro-economic phenomena.

"We now introduce the dynamic hypothesis that the rate at which w (micro wage level) changes is related to excess demand, and specifically, the greater is the proportionate disequilibrium the more rapidly will wages be changed" (Lipsey, 1960, p. 13). Various aspects of this question have been actively pursued by other economists.

For present purposes the import of the Lipsey paper is threefold. First it provides additional empirical support for Phillips' three original propositions: The rate of wage inflation is dependent on (1) the rate of unemployment, (2) changes in the rate of unemployment, and (3) the cost of living. Second the paper contains an early theoretical attempt to provide a micro-economic foundation to the observed trade-off phenomenon. However Lipsey's final set of regressions, summarized by Equation 1.9 above, suggests the

most important contribution of the study. By far the most important explanatory variable in this regression is the price inflation rate variable. Lipsey's circumspect discussion of these results suggests a recognition that the relationship may be open to a variety of interpretations. The most obvious is that the wage rate is just another price. Prices then are determined in an autoregressive fashion influenced only slightly by the unemployment rate. Nonetheless Lipsey chose to retain the Phillips thesis that increases in the general price level "cause" changes in the wage rate. Most subsequent analysts have chosen to follow his lead.

The body of Classical economic theory suggests a slightly different interpretation. The usual general equilibrium formulation of this system is homogeneous in nominal wages and prices. Quantities demanded and supplied depend on relative prices alone. The absolute price level must be determined from the outside. One might easily append to this a system of adjustment equations by which relative prices adjust gradually in response to nonzero levels of excess demand. In such a system the rate of change of real wages, that is the rate of change of money wages less the rate of price inflation, would depend on the excess demand for labor. Transposing the last term of Equation 1.9 to the left hand side gives an equation similar to the model above. The left hand side is, almost, the rate of change of real wages. One

might suggest that the unemployment rate provides a rough proxy for the excess demand for labor; when it is below normal and falling, excess demand is probably positive; when it is unusually high and rising, excess demand is negative. Thus we would expect to observe that increases in real wages occur during periods when the unemployment is low and falling and vice versa.

In the econometric search for a theory of price determination, however, such an approach has not been particularly popular.

The second major econometric approach to price level determination has become known as the New Inflation Theory. During the years of creeping inflation of the 1950's many economists came to accept a cost determined theory of the aggregate price level. This approach found expression in a large number of sectorial studies tending to support the thesis. Furthermore, a cost-push theory of price level changes came to be built into virtually all of the large scale quarterly U.S. econometric models developed during the 1960's. For once economists developed a theory of prices which corresponded to the layman's explanation.

The revival of a cost theory of the aggregate price level received an early formal treatment at the hands of Willard Thorp and Richard Quandt in The New Inflation (1959). The basic elements of the New Inflation theory are quite

simple. Cost increases are promulgated continually by various economic pressure groups attempting to better their positions vis-a-vis other groups through increased money These inflationary pressures together with public incomes. concern for the maintenance of low levels of unemployment and high rates of economic growth increase the probability that any increases in the general price level will be validated by Monetary and Fiscal policy actions. There is no direct conflict between the New Inflation theory and more traditional Monetary or Keynesian theories. The theory thus does not deny that cost-push may involve coincident increases in the money stock, fiscal expenditures and nominal income, particularly if real output is not allowed to decline. There is however a shift of the proximate causal factor from actions taken in the public sector to the institutional structure of the private sector. Hence public policy prescriptions which result from such a theory tend to stress programs designed to promote competition, increase labor mobility, and so on, and to deemphasize monetary and fiscal actions aimed only at affecting the level of aggregate demand.

There are several difficulties with a cost determined theory of price changes. It is, simply put, not a very elegant theory. Prices rise because prices rise. There is a definite risk of circularity in such an argument. It is a theory more of the mechanism through which price changes are

transmitted throughout the economy than of the cause of price changes. At best, cost push theory is a theory of autonomous price changes. However when attempts are made to interpret it as a theory of causation, the result, all too often, is a boogy-man price theory: Price increases result from "unwarrented" union wage demands; oil company "profiteering"; et cetera. If these charges are true, strict antitrust policy and wage and price controls could stop such inflationary pressures easily. Finally in the context of a cost push price theory the causal linkages between the factors affecting price and real output determination are somewhat indistinct. Thus it is difficult to integrate such a theory with, say, a standard Keynesian formulation.

In spite of these shortcomings it has become the dominant theory of price level determination. A goodly amount of empirical support has been mustered in support of the thesis.

A variety of approaches have been used to estimate cost push components of inflation. Wage push inflation is typically established by comparing the rate of change of money wages with the rate of change of labor productivity. It is assumed that any increases in wages not matched by productivity increases is inflationary. Such tests are simple to make, however they deal only with money wage movements, not with real wages, the proper variable in such a construct. There exist a plethora of such studies.

Phelps (1961) has suggested analysis of the distribution of relative income shares would provide an appropriate test between cost and demand caused inflation. An increase in wages relative to profits he argues would imply cost push inflation. Such an approach arises from a view of profits as a residual, not a cost. Neither of these approaches directly answers the question of how cost increases affect the general level of prices.

This question has been attacked by input-output analysis. It is in principle possible to trace the results of a cost increase occurring in one sector throughout the rest of the economy by using an input-output table (see for example Eckstein and Fromm, 1959). Such an exercise must assume all price increases are passed on in their entirety, and that no substitution takes place either during the intermediate production or final consumption stages.

The cost push approach of the New Inflation theorists became embodied in the major quarterly econometric models of the U.S. economy developed during the 1960's.¹ Each of these models contains a detailed financial sector. Linkages between this sector and the real sector are neo-Keynesian in

¹A review of the financial sectors of nine quarterly econometric models may be found in Carl Christ, 1971. The models he considered include the Warton Model (1967), the OEB Model (1966), the 1968 Michigan Model (1969), and various versions of both the Brookings Model and the FRB-MIT Models.

character with emphasis placed on the function of interest rates in the transmission mechanism. Prices are determined primarily by factor costs (wages, agricultural prices, import prices), secondarily by the general level of business activity (unemployment, capacity utilization, unfilled orders), together with a variable markup factor designed to capture trend, taxes, and so on.

Thus by the late 1960's the manifest "official" theory of the monetary sector consisted of a markup theory of prices and a financial sector - real sector linkage based on market rates of interest. The performance of this "official" theory in subsequent years was something less than exemplary.

Beginning in the last half of the 1960's, roughly coincident with the escalation of the Viet Nam war, and continuing well into the 1970's there was an apparent failure of monetary stabilization policy to curb inflation. High and rising market rates of interest were interpreted as indicators of a tight monetary policy by policy makers and model builders alike.

However econometric forecasts tended to understate levels of real output eventually realized. Meanwhile the money stock grew at an unprecidented rate. Gradually the rate of inflation rose to record levels.

It is the purpose of succeeding chapters to set forth a theory of price level and real output determination in which

changes in these aggregates result from internal, endogenous forces. Next within the context of such models the effects of various types of monetary and fiscal policies will be analyzed.

CHAPTER II

As a point of departure for later discussions consider the simplified aggregate macro economic model proposed by Milton Friedman (1970) for a closed economy with no government sector. Interpretation of the model is basically due to Friedman but differs however in some minor respects. <u>The</u> Friedman Model: a static equilibrium model

$$C/P = f(Y/P,R)$$
(2.1)

$$I/P = g(R) \tag{2.2}$$

$$Y/P = C/P + I/P$$
(2.3)

$$MD = P l(Y/P,R)$$
 (2.4)

$$MS = k(R)$$
 (2.5)

$$MD = MS \tag{2.6}$$

The first three equations describe relationships in the real sector of the economy; the last three define the behavior of the monetary sector. The two sectors are linked by the rate of interest and nominal income. The model as specified is under determined; there are seven unknowns (C, P, Y, R, I, MD, MS) and only six equations. Note that Y/P has been used for real income in the equations defining the model. A real income variable, y, is used later as a separate variable; the defining relationship y = Y/P is then counted as an independent equation. One additional relationship must be postulated among the variables to determine the

solution. For simplicity a number of important economic variables have been ignored, wealth, the capital stock, etc. Their inclusion would not change the basic nature of the argument which follows.

Equation 2.1 expresses the relationship between the real level of consumption demand on the one hand and real income and interest rates on the other. We may quite properly extend this to include all induced income dependent components of aggregate demand.

Equation 2.2, Keynes marginal efficiency of investment schedule, expresses the relationship between real investment demand and the rate of interest. Again we may extend this definition to include all components of demand which are interest elastic but independent of current income levels.

The income identity, Equation 2.3, defines an additivity requirement on the various endogenous components of the model. In this model there can be no distinction between desired and actual production or consumption decisions. Note also that the aggregate supply function has been suppressed here; supply is demand determined.

The "quantity theory" or "liquidity preference" Equation 2.4 expresses the relation between desired money balances, prices, income, and the rate of interest. One defining distinction between those who call themselves Keynesians and those who call themselves Monetarists has often been their

respective assumptions concerning the interest elasticity of money demand. The right hand side of this equation is assumed homogeneous of degree 1 in the price level.

Equation 2.5 is the money supply function. Money supply could be made exogenous if desired. This would reduce the model by one equation and one unknown. Finally Equation 2.6 defines equilibrium in the money market.

One further relationship must be specified to make the system solvable. In the naive quantity theory the added equation is

$$Y/P = y_0$$
, exogenous (2.7)

Real income is determined outside the system by the Walrasian, full employment, general equilibrium process. Changes in the money stock thus serve only to change the level of prices.

On the other hand in the naive income-expenditure theory the price level is determined outside the system.

$$P = P_{o}$$
, exogenous (2.8)

This approach leads to the familiar IS - LM analysis in which the first three equations serve to define the set of conditions necessary for equilibrium in the real sector; the last three define these conditions for the monetary sector. The two sectors together determine the joint solution for all variables. In either case the model is clearly an equilibrium construct. One interpretation is that it requires instantaneous adjustment by all variables from one equilibrium to another. The equilibria so reached are static unchanging levels of incomes, interest rates, and so on. The model ignores adjustment lags, expectations, unrealized desires and all other obvious attributes of the dynamic "real world". Economists of one persuasion or the other typically interpret their opponents' models in such a fashion.

The more charitable interpretation however is that such a model is truly comparative static in nature. It is assumed that underlying the model is a dynamic structure with stability properties such that the system will indeed tend to progress from one equilibrium toward the next in an orderly fashion and within a time frame short enough to make the comparison of the equilibria economically meaningful.

For many questions comparative static analysis is a proper and useful tool. It should be clear however that it cannot be used to answer essentially dynamic questions. The investigation of the process by which a change in nominal income is divided between real and price effects is clearly such a problem. Due to lags in the dynamic adjustment process, policies designed to affect real output in one period may in fact affect both prices and output in subsequent periods. For stabilization policy both current and

future effects are of critical interest. The static model must be modified to deal with these effects explicitly.

In recognition of this fundamental difficulty Milton Friedman has suggested (Friedman, 1970) the following dynamic approach to the problem of division.

$$\frac{dP}{dt} = f \left[\frac{dY}{dt}, \frac{dP^*}{dt}, \frac{dy^*}{dt}, y, y^* \right]$$
(2.9)

$$\frac{dy}{dt} = g \begin{bmatrix} \frac{dY}{dt}, \frac{dP^*}{dt}, \frac{dy^*}{dt}, y, y^* \end{bmatrix}$$
(2.10)

Where asterisks indicate anticipated values and lower case y is real income, Y/P. The system must be consistent at each point in time with the identity

$$Y = Py.$$
 (2.11)

This implies the rate of change identity

$$Y'/Y = P'/P + Y'/Y$$
 (2.12)

As an illustration he suggests the relationships

$$\frac{d \log P}{dt} = \frac{d \log P^*}{dt} + \alpha \left(\frac{d \log Y}{dt} - \frac{d \log Y^*}{dt} \right) + \gamma (\log y - \log y^*)$$
(2.13)

$$\frac{d \log y}{dt} = \frac{d \log y^*}{dt} + (1-\alpha) \left(\frac{d \log Y}{dt} - \frac{d \log Y^*}{dt} \right)$$
$$-\gamma (\log y - \log y^*) \qquad (2.14)$$

The rate of change of prices is dependent on the anticipated price change, the discrepancy between actual and anticipated rates of growth of nominal income and the logarithm of the ratio of actual to anticipated real income. Similarly the rate of change of real income depends on its anticipated rate of change adjusted as before for any discrepancy between actual and anticipated rates of change of nominal income and actual and anticipated levels of real income. Equations 2.13 and 2.14 sum to 2.12.

Thus Friedman has added three dynamic relationships 2.12, 2.13, and 2.14 to the system. Ignoring the starred variables for the moment, it would appear at first glance that the system is now over determined. The static model, including y as a variable and the decomposition equation y = Y/P as an equation, contains 8 unknowns and 7 independent equations, and hence has one degree of freedom. Only one additional independent relationship may be added to close the model. Note however that 2.12 is not independent of 2.11, but is simply the dynamic analogue. Further, by construction 2.13 and 2.14 add up to 2.12, a linear dependence. The three dynamic equations provide only a single relationship independent of the original model. Therefore, still ignoring the starred variables, the system now has 8 unknowns and 8 independent equations (2.1 through 2.6, 2.11, and either 2.13 or 2.14).

In addition there are three "anticipation" variables (Y*, P*, y*). Jumping ahead, these are defined by Equations 2.15, 2.17, 2.18, 2.19. These four equations contain three independent relationships by construction. These relationships may be differentiated to obtain the dynamic analogues; however as before, in Equations 2.11 and 2.12, this does not provide any new information. The anticipation subsystem thus contains 3 unknowns and 3 independent relationships, and is completely determined. Hence the rationale of the previous paragraph was not destroyed by ignoring these variables. The complete system now contains 11 unknowns and 11 independent relationships.

The anticipation variables are linked by the identity

$$Y^* = P^* Y^*$$
 (2.15)

It is of interest to postulate a relationship between the anticipated levels of these variables and observed past levels of economic activity. It is often suggested that a weighted average of past levels of actual income may serve as a proxy for anticipated, or permanent income.

Suppose we choose as a special case the geometrically declining weight structure

$$w(\tau) = We^{-\delta(t-\tau)} \qquad \delta > 0$$

$$\tau \varepsilon [t-\theta, t] \qquad (2.16)$$

with W chosen such that

$$f w(\tau) d\tau = 1$$

t-0

Thus $W = \delta/1 - e^{-\delta \theta}$ where θ is the length of time over which expectations are formed and the parameter δ determines the shape of the weight structure: The larger the value of δ the greater is the weight placed on more recent experience. Thus

$$Y_{t}^{*} = \int_{t-\theta_{1}}^{t} w_{1}(\tau) Y(\tau) d\tau \qquad (2.17)$$

$$P_{t}^{*} = \int_{t-\theta_{2}}^{t} w_{2}(\tau)P(\tau)d\tau \qquad (2.18)$$

$$y_{t}^{*} = \int_{t-\theta_{3}}^{t} w_{3}(\tau)y(\tau)d\tau \qquad (2.19)$$

where

$$w_{i}(\tau) = \frac{\delta_{i}}{1-e} e^{-\delta_{i}(t-\tau)}$$

and

 $Y^{*}(t) = P^{*}(t)y^{*}(t)$

Friedman assumes, Equation 2.11, that nominal income is divisible into the product of its component indices, price and real output. In a similar vein we have suggested a similar relationship for a rational system of expectations, Equation 2.15. For this identity to be satisfied at all points

$$Y^*'/Y^* = P^*'/P^* + Y^*/Y^*$$
 (2.20)

where

$$\frac{\mathbf{Y}^{\star \mathbf{v}}}{\mathbf{Y}^{\star}} = \frac{\begin{array}{c} t\\ f\\ \mathbf{t}^{-\theta} \\ t\\ t\\ f\\ \mathbf{t}^{-\theta} \\ t\\ t \\ t \\ t \\ t \\ t \\ \theta \\ t \end{array}} (\mathbf{1}) \mathbf{Y}(\tau) d\tau \qquad (2.21)$$

However Equation 2.21 may be decomposed by reference to identity 2.15

$$\frac{\mathbf{Y}^{*}}{\mathbf{Y}^{*}} = \frac{\int_{t-\theta_{1}}^{f} \mathbf{w}_{1}(\mathbf{P}^{*}\mathbf{y} + \mathbf{P}\mathbf{y}^{*}) dt}{\mathbf{Y}^{*}}$$

which must equal the RHS of 2.20. Thus

$$= y^{*} \int_{t-\theta_{2}}^{t} w_{2}^{P'd\tau} + P^{*} \int_{t-\theta_{3}}^{t} w_{3}^{Y'd\tau} \qquad (2.22)$$

A relationship which must hold at all points.

Consider $y^{\dagger} = 0$ over an interval long enough that $y^{*} = y$. Then 2.22 reduces to

$$t \\ f \\ t - \theta_1 \\ t - \theta_1 \\ t - \theta_2 \\ t - \theta_2 \\ t - \theta_2 \\ t - \theta_2$$

For this to hold over all possible time paths of p' this must be an identity. Hence $\theta_1 = \theta_2$ and $w_1(\tau) \equiv w_2(\tau)$. By a similar argument $\theta_1 = \theta_3$ and $w_1(\tau) = w_3(\tau)$.

Thus we arrive at a necessary requirement for the consistency of such a system of expectation formulation: Expectations about nominal income, prices, and real income, when based on geometrically weighted averages of past observations, must be formed in an identical manner if these expectations are to satisfy the identity $Y^* = P^*y^*$. By a similar method of proof an identical requirement exists for other strategies of expectation formation. It is important to note that this precludes interpretation of y^* as the level of long run full employment real output. It will be shown later in this chapter that reference to such a full employment benchmark is necessary to force long run money neutrality and complete price adjustment in a model such as this.

The partially reduced form of the static system may be expressed as a single equation containing any three of the eight variables of the static subsystem. Since the focus here is on the effects of monetary policy the system may be solved in terms of the observable variables Y, P, and M. This is the approach taken in Equation 2.29. Alternatively, making use of the decomposition identity Y = Py, we may solve for real income y in terms of P and M. Any such equation will be referred to as a static equilibrium condition. Similarly, using the remaining equations of the model, one may arrive at a partially reduced dynamic equation defining the division mechanism. For example using 2.13 as a starting point

$$\frac{P'}{P} = \frac{t - \theta}{t} + \frac{Y'}{Y} - \alpha \frac{t - \theta}{t} + \frac{Y'}{t - \theta}$$

$$+ \gamma \ln \left(\frac{Y}{t} \right) - \gamma \ln \left(\frac{P}{t} \right)$$
(2.23)
$$+ \gamma \ln \left(\frac{T}{t} \right) + \gamma \ln \left(\frac{P}{t} \right)$$
(2.23)

These two equations, one defining static, period by period equilibrium, the other defining period to period adjustment, are sufficient to completely determine the future behavior of the system.

To facilitate this discussion we will consider the following simplified version of the general macro model developed thus far. A simple explicit version of the model will be used in subsequent discussions. The money supply Equation 2.5 will be changed to make the money stock exogenous. The central bank determines the nominal stock of money while the real value of this stock and the terms under which it is held are determined in the private sector. Thus the system is one which can be controlled by monetary policy alone. It is the purpose of the next chapter to investigate various
schemes of the monetary control of such a system. Further the system will be described by difference rather than differential equations for computational simplicity.

Equilibrium in the commodity sector is described by three equations.

$$Y = C + I \tag{2.24}$$

$$C = b_{C} Y$$
 $0 < b_{C} < 1$ (2.25)

$$I = P \cdot b_{\underline{I}} \cdot r^{\underline{EI}} \quad \underline{EI} < 0$$
 (2.26)

If desired these may be solved for the Hicksian IS curve

$$\mathbf{r} = \left(\frac{\mathbf{Y}}{\mathbf{P}} \cdot \frac{\mathbf{1} - \mathbf{b}_{\mathbf{c}}}{\mathbf{b}_{\mathbf{I}}}\right)^{\frac{1}{\mathbf{E}\mathbf{I}}}$$
(2.27)

Money market equilibrium is defined by the equation, Hick's LM curve,

$$M b_{M} r^{EM} = Y EM > 0$$
 (2.28)

The money stock is viewed as an exogenous control variable. One may solve for nominal income in terms of the price level and the stock of money.

Let

$$A = b_{M} \frac{EI}{EI-EM} \left(\frac{b_{I}}{1-b_{C}}\right)^{\frac{-EM}{EI-EM}}$$

Then

$$Y = A \cdot p^{\frac{-EM}{EI-EM}} \cdot M^{\frac{EI}{EI-EM}}$$
(2.29)

The system is homogeneous of degree 0 in nominal income, prices, and the money stock.

$$\lambda \cdot Y = \lambda \cdot A \cdot P^{\frac{-EM}{EI-EM}} \cdot Y^{\frac{EI}{EI-EM}} = A \cdot (\lambda \cdot P)^{\frac{-EM}{EI-EM}} \cdot (\lambda \cdot M)^{\frac{EI}{EI-EM}}$$

Differentiating the logarithm of Equation 2.29 gives

$$\frac{Y'}{Y} = \frac{-EM}{EI-EM} \cdot \frac{P'}{P} + \frac{EI}{EI-EM} \cdot \frac{M'}{M}$$
(2.30)

Note that in general the rate of growth of nominal income is not expected to equal the rate of growth of the money stock. This is due to the fact that changes in M not matched by proportionate price changes lead to changes in money market interest rates and hence to changes in the income velocity of money. However if the interest elasticity of money demand, -EM, is zero, velocity is constant and Y'/Y always equals M'/M. In such a case the system displays the so called Classical Dicotomy with the interest rate determined solely in the real sector.

If an arbitrary set of values is chosen for the parameters of the system, the response of the system through time may be observed for various growth rates of the money stock. Table 1 contains the results of such an exercise for a system with a constant price level and a rate of money stock

Table 2.1.

The static model	with increasing	money stock (An
arbitrary set of	parameter values	was selected for
use in all simula	tions used in th	is study. Values
of these paramete	ers may be found	in Appendix A.)

С	I	v	Y	% ∆¥	R
194,704	21.634	2.16	216.337		6.24
198.579	22.064	2.14	220,643	1.990	5,94
202.531	22.503	2.12	225.034	1.990	5.65
206.561	22.951	2.10	229.512	1.990	5.38
210.672	23.408	2.08	234.080	1.990	5.12
214.865	23.874	2.06	238.738	1.990	4,88
219.141	24,349	2.04	243.490	1.990	4.64
223.502	24.834	2.02	248.335	1.990	4.42
227.950	25.328	2.00	253.278	1.990	4.21
232.436	25,832	1.98	258.318	1.990	4.01
237.113	26.346	1.96	263.459	1.990	3.81
241.832	26.870	1.94	268.702	1.990	3.63
246.645	27.405	1.92	274.050	1.990	3.46
251.553	27,950	1.90	279.504	1.990	3.29
256,560	28,507	1.88	285.066	1.990	3.13
261.665	29.074	1.87	290.739	1.990	2,98
266.873	29.653	1.85	296.526	1.990	2.34
272.184	30.243	1.83	302.427	1.990	2.70
277.601	30.845	1.81	308.445	1.990	2.57
283.126	31.458	1.79	314.584	1.990	2.45

growth, RM, of 3%. A listing of parameter values used in this and all subsequent simulations may be found in Appendix A.

The standard Income Expenditure model is explicitly a short run formulation. Its implicit policy implications provide a general theoretical guide for the appropriate conduct of short period monetary policy. However the model is not well suited for discussion and analysis of long term monetary policy for a growing economy. Its principal shortcoming lies in the nature of the investment function, the equation governing the behavior of the income-independent components of aggregate demand.

It is evident from Table 1 that a steady rate of monetary growth leads to a steady rate of growth of real income. This increase is effected however by a decline in the rate of interest. Only a decline in the interest rate can induce the increase in the level of investment demand necessary to match increased savings which results from income growth. At the same time interest rate declines induce a secular decrease in the velocity of money corresponding to the reduced opportunity cost of holding money balances.

In a growing economy one might expect to observe a secular increase in real investment through time if interest rates remain constant, reflecting the desire to devote a constant proportion of an ever increasing real output to capital accumulation. To allow for this possibility formally the investment function may be altered slightly. Assume a secular increase in real income at a constant rate, RYf. Replace Equation 2.26 by

$$I = P \cdot b_{I} \cdot \prod_{i=1}^{t} \{(1 + RBI_{i}) \cdot (1 + RYf)\}r^{EA} \quad (2.31)$$

For the moment let $RBI_i = 0$ for all i; this factor will be considered later. Exogenous real investment grows at a rate RVf for any fixed level of the interest rate. For convenience this will be referred to as the "natural" rate of investment growth. At any particular point in time marginal investment activities are inversely related to interest rate. Through this mechanism monetary policy is capable of influencing the ratio of investment to real output. A simulation of the model with this modification is provided in Table 2. The static equilibrium condition with this modification becomes

 $Y_{t} = a\{b_{i} | i | (1 + RBI_{i}) (1 + RYf)]\} \xrightarrow{-EM}_{EI-EM} p_{t} \xrightarrow{-EM}_{H} M_{t} \xrightarrow{EI}_{EI-EM}$

where

$$a = b_{M}^{\frac{EI}{EI-EM}} \left(\frac{1}{1-b_{c}}\right)^{\frac{-EM}{EI-EM}}$$
(2.32)

Note that with constant prices if the money stock grows at rate RM equal to RYf, the rate of exogenous demand increase, nominal income will also grow at rate RM. The system will be stationary in the sense that velocity and interest rates will be constant through time.

With this modification a constant rate of monetary growth at 3%, the natural rate of investment growth, yields a constant rate of real income increase at 3%, with constant velocity and interest rates. These results are somewhat more

		Ŧ			
С	I	v	Y	₹∆¥	R
194.704	21.634	2.16	216.337		6.24
200.545	22.283	2.16	222.823	3.000	6.24
206.561	22.951	2.16	229.512	3.000	6.24
212.758	23,640	2.16	236.398	3.000	6.24
219.141	24.349	2.16	243.490	3.000	6.24
225.715	25.079	2,16	250.794	3.000	6.24
232.436	25.832	2.16	258.318	3.000	6.24
239.461	26,607	2,16	266.068	3.000	6.24
246.645	27.405	2.16	274.050	3.000	6.24
254.044	28.227	2.16	282.271	3.000	6.24
261.665	29.074	2.16	290,739	3.000	6.24
269.515	29.946	2.16	299.462	3.000	6.24
277.601	30.845	2.16	308.445	3.000	6.24
285,929	31,770	2.16	317.699	3.000	6.24
294.507	32.723	2.16	327.230	3.000	6.24
303.342	33.705	2.16	337.047	3.000	6.24
312,442	34,716	2.16	347.158	3.000	6.24
321.816	35.757	2.16	357.573	3,000	6.24
331.470	36.830	2.16	368.300	3,000	6.24
341.414	37.935	2.16	379.349	3.000	6.24

Table 2.2. The static model with Friedman Rule monetary growth (RBI; = 0, RYf = .03, RM = .03)

appealing intuitively.

We may now define RBI_i as the cyclically varying component of demand and let RYf represent the long term growth rate of the economy. With this modification the period equilibrium sector of the model may be dealt with either as a description of a stationary or a growing economy by varying the rate of long term growth RYf. Further business cycles could be generated by varying RBI_i through time. This represents, ceteris paribus, a cyclic variation in the proportion of real output which is channeled into investment activities as well as any other cyclic compents of exogenous demand. Finally by dividing the period reduced form equation by (1 + RYf), where RYf is the full employment or long term trend rate of growth of the economy, it is possible to isolate the cyclic behavior of the economy from its long term trend behavior and to analyze the cyclic behavior of a growing economy as though it were a no growth system.

The reduced form adjustment equation, 2.23, can be dealt with most conveniently with a difference equation approximation. Expectations about the future are assumed to be based primarily on current period expectations modified in light of actual current developments. A simple form of such a mechanism is

$$P_{t}^{\star} = P_{t-1}^{\star} \cdot \left(\frac{P_{t-1}}{P_{t-2}}\right)$$

Future expectations are formed by simple extrapolation of current expectations.

Unfortunately such a simple formulation causes the rate of change of expectations to equal the lagged observed rate of change of the variable in question; if prices are constant through time, expected price is constant also--even if expected and actual prices are not equal.

To avoid this possibility a learning factor term may be introduced to gradually force these terms together in such a situation. Thus the expectation equations become

$$P_{t}^{\star} = P_{t-1}^{\star} \cdot \left(\frac{P_{t-1}}{P_{t-2}}\right) \cdot \left(\frac{P_{t-1}}{P_{t-1}^{\star}}\right)^{EW}$$

$$Y_{t}^{\star} = Y_{t-1}^{\star} \cdot \left(\frac{Y_{t-1}}{Y_{t-2}}\right) \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{\star}}\right)^{EW}$$
(2.33)

$$t = 1, N = EW < 1$$

The last two terms of Friedman's price adjustment equation, 2.23, serve as an accelerator dependent on deviations between actual real income and anticipated real income. Because of their peculiar form we will ignore them for the present. The remainder of that equation is merely the derivative of a power function in P*, Y, and Y*, that is

$$P_{t} = P_{t}^{*} \cdot \begin{pmatrix} Y_{t} \\ \overline{Y}_{t}^{*} \end{pmatrix} EA \qquad (2.34)$$

Similarly

$$y_t = y_t^* \cdot \left(\frac{Y_t}{Y_t^*}\right)^{1-EA}$$

and

$$P_{t}Y_{t} = P_{t}^{\star} \cdot Y_{t}^{\star} \cdot \left(\frac{Y_{t}}{Y_{t}^{\star}}\right) = Y_{t}$$

Current price is responsive to monetary policy only indirectly through the relationship between the money stock and nominal income, the static equilibrium condition, 2.32.

Simultaneous solution of 2.32 and 2.34 yields

$$P_{t}^{\star} = P_{t-1}^{\star} \cdot \frac{P_{t-1}}{P_{t-2}} \cdot \left(\frac{P_{t-1}}{P_{t-1}^{\star}}\right)^{EW}$$

$$\mathbf{Y}_{t}^{\star} = \mathbf{Y}_{t-1}^{\star} \cdot \frac{\mathbf{Y}_{t-1}}{\mathbf{Y}_{t-2}} \cdot \left(\frac{\mathbf{Y}_{t-1}}{\mathbf{Y}_{t-1}^{\star}}\right)^{\mathrm{EW}}$$

$$P_{t} = (a^{EA}) \frac{EI-EM}{D} \cdot \{b_{1} \cdot \prod_{i=1}^{t} (1+RBI_{i}) (1+RYf)\}^{D} \cdot \frac{P_{t}^{*}}{Y_{t}^{*EA}} \frac{EI-EM}{D} \cdot M_{t}^{\frac{EA \cdot EI}{D}}$$

$$Y_{t} = a \xrightarrow{EI-EA}_{D} \cdot \{b_{I} \cdot \prod_{i=1}^{t} (1+RBI_{i}) (1+RYf)\} \cdot \left(\frac{P_{t}^{*}}{Y_{t}^{*EA}}\right) \xrightarrow{-EM}_{D} \cdot M_{t} \xrightarrow{EI}_{D}$$
(2.35)

where
$$a = b_M \frac{EI}{EI - EM} \frac{1}{1 - b_A} \frac{-EM}{EI - EM}$$

$$D = EI - (1 - EA) EM$$

As an empirical observation the price level does not appear to be significantly influenced by current monetary policy. Indeed the evidence suggests that price movements are largely auto generating in nature. Accordingly the model is not damaged theoretically by modifying the system still further by lagging the income accelerator term in the price Equation 2.34. This simplifies computation appreciably by introducing a degree of recursiveness into the model. Thus

$$P_{t} = P_{t}^{*} \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{*}}\right)^{EA}$$
(2.36)

This leads to the modified set of recursive equations

$$P_{t}^{\star} = P_{t-1}^{\star} \cdot \left(\frac{P_{t-1}}{P_{t-2}}\right) \cdot \left(\frac{P_{t-1}}{P_{t-1}^{\star}}\right)^{EW}$$

$$Y_{t}^{\star} = Y_{t-1}^{\star} \cdot \left(\frac{Y_{t-1}}{Y_{t-2}}\right) \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{\star}}\right)^{EW}$$

$$P_{t} = P_{t}^{\star} \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{\star}}\right)^{EA}$$

$$Y_{t} = a \cdot \{b_{I} \cdot \prod_{i=1}^{t} (1+RBI_{i})(1+RYf)\} \xrightarrow{-EM} \cdot P_{t} \xrightarrow{-EM} \cdot M_{t} \xrightarrow{EI} (2.37)$$

These may be solved sequentially.

The model was simulated under both formulations with no qualitative differences in behavior save for a slight extension of the price adjustment lags. A simulation of the model over 30 periods is provided in Table 3. Exogenous demand was allowed to grow at 3%. M was increased at 3%, 6%,

Y/P	Y	% Δ¥	P	% ∆₽	¥*	P*	М	v	R
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
222.83	222.83	3.0000	1.0000	0.000	222.83	1.0000	103.00	2.16	6.24
229.51	229.51	3.0000	1.0000	0.000	229.51	1.0000	106.09	2.16	6.24
236.40	236.40	3.0000	1.0000	0.000	236.40	1.0000	109.27	2.16	6.24
243.49	243.49	3.0000	1.0000	0.000	243.49	1.0000	112.55	2.16	6.24
255.64	255.64	4.9904	1.0000	0.000	250.79	1.0000	119.30	2,14	5.95
267.37	268.91	5.1915	1.0058	0.575	263.81	1.0000	126.46	2.13	5.73
279.54	282.93	5.2117	1.0121	0.633	278.04	1.0063	134.05	2.11	5.52
292.24	297.68	5.2137	1,0186	0.639	293.04	1.0133	142.09	2.09	5.31
305.53	313.20	5.2139	1.0251	0.639	308.81	1.0203	150.62	2.08	5.12
309.29	319.08	1.8787	1.0317	0.639	325.37	1.0273	152.12	2.10	5.35
315.12	324,03	1,5511	1.0283	-0.327	330.83	1.0343	153.65	2.11	5.49
321.26	328.95	1.5184	1.0239	-0.424	335.27	1.0303	155.18	2.12	5.64
327.55	333.94	1.5151	1.0195	-0.433	339.71	1.0253	156.73	2.13	5.78
333.97	338,99	1.5148	1.0151	-0.434	344.27	1.0203	158.30	2.14	5.93
347.22	350.91	3.5151	1.0106	-0.434	348.94	1.0153	164.63	2.13	5.79
359.58	363,95	3.7173	1.0122	0.149	361.41	1.0104	171.22	2.13	5.72
372.25	377.56	3.7375	1.0143	0.208	375.11	1.0121	178.07	2.12	5.64
385.34	391.68	3.7396	1.0164	0.214	389.38	1.0145	185.19	2.11	5.57
398.90	406.32	3.7398	1.0186	0.214	404.18	1.0168	192.60	2.11	5.50
410.28	418.81	3.0737	1.0208	0.214	419.52	1.0192	1 98. 38	2.11	5.52
422.53	431.41	3.0073	1.0210	0.021	432.34	1.0215	204.33	2.11	5.53
435.20	444.35	3.0007	1.0210	0.002	445.25	1.0217	210.46	2.11	5.53
448.25	457.68	3.0000	1.0210	0.000	458.51	1.0217	216.77	2.11	5.53
461.70	471.41	3.0000	1.0210	0.000	472.18	1.0216	223.27	2.11	5.53
475.55	485.56	3.0000	1.0210	0.000	486.27	1.0215	229.97	2.11	5.53
489.82	500.12	3.0000	1.0210	0.000	500.78	1.0215	236.87	2.11	5.53
504.51	515.13	3.0000	1.0210	0.000	515.74	1.0214	243.98	2.11	5.53
519.65	530.58	3.0000	1.0210	0.000	531.15	1.0214	251.30	2.11	5.53
535.24	546.50	3.0000	1.0210	0.000	547.02	1.0214	258.84	2.11	5.53

Table 2.3. Behavior of the Friedman model in a growing economy

1%, and 4% for five periods each, then at 3% for the remaining 10 periods. Initial conditions consistent with a constant price level and fulfilled expectations were specified.

The model has a number of important stability properties. Qualitative behavior of the system was not altered in simulations undertaken with a variety of different parameter values -- so long as each of the exponential parameters in the adjustment equations was chosen between zero and one. Whenever the rate of monetary growth is set equal to the rate of increase of exogenous demand the model stabilizes at a constant rate of growth of real output and prices. However this constant rate of growth of prices need not always be zero; if it is nonzero, velocity and interest rates will not be constant through time. Real output will decline to compensate for price increases.

The system is theoretically perverse. One of the principal tenets of the monetary theorists is that in the long run monetary movements have no influence on the real sector. In this model with a constant rate of interest real output grows at the same rate as exogenous demand. With monetary growth at this rate prices should adjust to a constant level.

In some cases they do. Even in such cases however the burden of adjustment to a monetary disturbance does not fall entirely on price changes. Part of the adjustment is borne

by permanent changes in interest rates and nominal velocity. The model does not display the classical long run trait of money neutrality.

To put it another way consider a no growth economy, i.e. one in which exogenous demand is constant through time. A 10% increase in the money stock does not imply a 10% increase in prices. At the new equilibrium prices will have increased somewhat less than 10%, interest rates will have declined, and real output increased.

Another principal tenet of monetary theorists is that a rate of monetary growth equal to the rate of increase in real output will lead to long run price stability. Under certain conditions the model displays this characteristic, under others it does not. The difference results from choice of initial conditions used to start up the model.

Consider the system summarized by Equations 2.32 through 2.34. Equation 2.34 may be used to eliminate P* from the first expression in 2.33. The second expression in 2.33 may then be used to eliminate all Y* terms. This yields

$$\left(\frac{\mathbf{P}_{t} \cdot \mathbf{P}_{t-2}}{\mathbf{P}_{t-1}^{2}} \right) = \left(\frac{\mathbf{Y}_{t} \cdot \mathbf{Y}_{t-2}}{\mathbf{Y}_{t-1}^{2}} \right)^{\mathrm{EA}}$$

Equation 2.32 is of the form

 $Y_t = K_t P_t^{\delta} M_t^{1-\delta}$

for any constant growth rate of exogenous demand the K's will cancel on substitution. Thus

$$\begin{pmatrix} P_{t} \cdot P_{t-2} \\ P_{t-1}^{2} \end{pmatrix} = \begin{pmatrix} M_{t} \cdot M_{t-2} \\ M_{t-1}^{2} \end{pmatrix}^{\frac{EA(1-\delta)}{1-\delta EA}}$$

For every constant rate of growth of the money stock the RHS equals one. Then

$$\frac{P_{t}}{P_{t-1}} = \frac{P_{t-1}}{P_{t-2}}$$
,

a second order difference equation completely determined by the two initial conditions and independent of the rate of monetary growth so long as it remains constant. Given unfavorable initial conditions it is not possible to achieve both constant prices and a rate of monetary growth equal to the long term rate of growth in exogenous demand. For example in a no growth system with constant money stock but initial prices nonconstant, prices will continue to grow forever at the initial rate.

Utilizing the lagged adjustment price Equation 2.36 in place of 2.34 yields similar assymptotic results.

Solution of 2.33 and 2.36 gives

$$\left(\frac{\overset{\mathbf{P}_{t}}{\mathbf{p}_{t-1}}\overset{\mathbf{P}_{t-2}}{\mathbf{p}_{t-1}}\right) = \left(\frac{\overset{\mathbf{P}_{t-1}}{\mathbf{p}_{t-2}}\overset{\mathbf{P}_{t-3}}{\mathbf{p}_{t-2}}\right)^{\delta \text{EA}} \cdot \left(\frac{\overset{\mathbf{M}_{t-1}}{\mathbf{m}_{t-3}}\overset{\mathbf{M}_{t-3}}{\mathbf{m}_{t-2}}\right)^{(1-\delta) \text{EA}}$$

Again for a constant rate of monetary growth, any steady rate of price change specified in initial conditions, zero or nonzero, will be perpetuated forever. Thus if $P_1 = P_0(1+k)$ and $P_2 = P_1(1+k)$, then prices will grow at rate k for all time with any steady rate of monetary growth.

This system is even more complex however; the three initial conditions need not lie on a steady rate growth path. Let $Z_t = P_t P_{t-2}/P_{t-1}^2$; then $Z_2 = P_2 P_0/P_1^2$, defined by the boundary conditions. The ratio of adjacent period prices under constant monetary growth is given by

$$\frac{P_{t}}{P_{t-1}} = z_{2}^{i=1} \cdot \frac{P_{2}}{P_{1}}, \text{ for } t > 2.$$

Note that both δ and EA are positive but less than 1. Thus the RHS of this expression is bounded by the function

$$\frac{P_{t}}{P_{t-1}} = Z_{2}^{\frac{\delta EA}{1-\delta EA}} \cdot \frac{P_{2}}{P_{1}}, \text{ a constant.}$$

The trajectary of prices is asymptotic to some steady rate inflation path dependent on initial conditions.

The model thus far displays all of the short run properties usually associated with Income-Expenditure models. In addition it possesses some long run properties associated with the monetarist position. One slight modification of the system 2.37 yields a system consistent with all the principal properties of both approaches.

The literature in economics is replete with references to the full employment level of output. Reasonably this full employment level is defined not as the output obtainable by maximum utilization of all resources, but as that level obtainable by fully utilizing all resources at normal levels. Demand inflation theorists have generally argued that levels of aggregate demand in excess of such a full employment level tend to generate pressure for price increases. Levels of aggregate demand below this level create pressures for price declines. These pressures are assumed to vary depending on the degree of excess demand exhibited by the economy. Accordingly the price equation will be modified to allow for this tendency.

Define YF as the level of full employment income at nominal prices and RYf as the rate of growth of the level of full employment, where the level of real full employment is determined exogenously as the result of, say, a Walrasian general equilibrium process. In a more complex model the level of real full employment output would depend, at a minimum, on past levels of investment. Exogeneity is assumed here as a simplification. Full employment real output is assumed to grow at the same rate as the long term trend in exogenous demand, a familiar result in capital theory

(Sengupta, 1970; Shell, 1967). Then

$$Yf_t = \left(\frac{Yf_{t-1}}{P_{t-1}}\right) \cdot (1 + RYf) \cdot P_t$$

Using this full employment level of real output as a reference point, a "Keynesian" price adjustment mechanism may be introduced into the model (Keynes, 1936, Chapter 21). It is assumed that there exists a positive relationship between the level of real output and the rate of change of prices. Such a macroeconomic relationship has a firm microeconomic foundation. In the short run as real output is increased from low to high levels relative to the normal full employment level of real output, a number of factors combine to increase unit production costs. As more variable imputs are applied to fixed factors of production in many industries, declining marginal productivity may tend to drive up unit costs. Further increased factor demand tends to increase the bargaining power of primary and intermediate production factors, leading to higher factor imput costs. Finally as production capacity is approached in certain industries bottlenecks and shortages may develop. These and similar forces tend to lead producers to attempt to increase output prices. At the same time increases in aggregate income generated by increased factor employment lead to demand shifts for individual commodities. The net result may well be

gradual increases in the aggregate price level.

It is assumed that at a constant price level there exists some level of real factor employment which is considered normal in some sense, and that this level of employment does not generate pressures for changes in the aggregate price level on balance. However deivations from this level create inflationary pressures, positive or negative, which tend to return the system to the normal full employment level. It is recognized that this mechanism is but one of many forces which act in conjunction to determine the average level of prices.

The importance of this modification for the present model lies in the fact that it forces a complete price adjustment in the long run to disturbances which result from purely monetary forces. It is the linkage between real output and the price level which adds to the behavior of the system the properties of (1) long run money neutrality in response to a one time monetary disturbance, and (2) long run price stability properties which are independent of the initial rate of change in prices. In all cases a rate of growth of the money stock equal to the rate of growth of exogenous demand and real full employment output will result in assymptotically stable prices.

Equation 2.34 may be modified by adding the ratio (Y_{+}/Yf_{+}) to the RHS. To allow partial price response in any

given period this ratio is raised to a positive power EF < 1. Thus 2.34 becomes

$$P_{t} = P_{t}^{*} \cdot \left(\frac{Y_{t}}{Y_{t}^{*}}\right)^{EA} \cdot \left(\frac{Y_{t}}{Yf_{t}}\right)^{EF}$$
(2.38)

Lagging the income dependent effects one period as before, Equation 2.36, yields

$$P_{t} = P_{t}^{*} \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{*}}\right)^{EA} \cdot \left(\frac{Y_{t-1}}{Y_{t-1}}\right)^{EF}$$
(2.39)

Table 4 contains a simulation of the model 2.37 with this single modification. Note in the final 10 period segment that, even though the system has not completely adjusted, within acceptable tolerances velocity and the rate of interest are approaching what they were in the first 5 period block. Prices apparently are absorbing the entire brunt of adjustment.

To characterize the behavior of the system still further consider the effect of a permanent 20% increase of the money stock in a no growth economy. Results of such an exercise are shown in Table 5.

Tables 4 and 5 serve to demonstrate the essential and unique properties of the model. First, the basic properties of the model do not depend on the long term growth rate of the economy; the economy may be growing through time as in Table 4 or static as in Table 5. It is possible then to

Y/P	Y	% Δ¥	P	% ∆P	¥*	P*	М	v	R
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
222.83	222,83	2,9999	1.0000	0.000	222.83	1.0000	103.00	2.16	6.24
229.51	229.51	2.9999	1.0000	0.000	229.51	1.0000	106.09	2.16	6.24
236.40	236.40	3.0000	1.0000	0.000	236.40	1.0000	109.27	2.16	6.24
243.49	243.49	3.0000	1.0000	0.000	243.49	1.0000	112.55	2.16	6.24
255.64	255.64	4.9904	1.0000	0.000	250,79	1.0000	119.30	2.14	5.95
265.67	269.77	5.5277	1.0154	1.543	263.81	1.0000	126.46	2.13	5.82
274.82	285.34	5,7726	1.0383	2.251	279.02	1.0170	134.05	2.13	5.76
283.48	302.24	5,9224	1.0662	2.686	295.79	1.0420	142.09	2.13	5.73
291.87	320.44	6.0204	1.0979	2.972	313.98	1.0725	150.62	2.13	5.74
290.64	329.16	2.7205	1.1325	3,155	333.56	1.1070	152.12	2.16	6.25
294.22	335.34	1.8781	1.1397	0.638	342.18	1.1445	153.65	2,18	6.52
300.07	340.37	1.5004	1.1343	-0.477	347.91	1.1513	155.18	2.19	6.68
307.45	344.68	1,2659	1.1211	-1.165	352.35	1.1441	156.73	2.20	6.77
316.00	348.50	1.1074	1.1028	-1.628	356.03	1.1285	158.30	2.20	6.81
331.89	358.92	2.9908	1.0814	-1.940	359.20	1.1076	164.63	2.18	6.19
345.45	371.33	3.4567	1.0749	-0.603	369.92	1.0835	171.22	2.17	6.32
358.17	384.91	3.6577	1,0747	-0.022	382,85	1.0761	178.07	2.16	6.21
370.45	399.47	3.7833	1.0783	0.340	397.07	1.0757	185.19	2.16	. 6.15
382.52	414.93	3.8709	1.0847	0.595	412.34	1.0796	192.60	2.15	6.11
391.97	428.48	3.2655	1.0932	0.775	428.57	1.0866	198.38	2.16	6.19
402.70	441.90	3.1309	1.0973	0.381	442.55	1.0957	204.33	2.16	6.23
414.15	455,50	3.0793	1.0999	0.231	456.34	1.1000	210.46	2.16	6.25
426.17	269.39	3.0487	1.1014	0.142	470.31	1.1025	216.77	2.17	6.27
438.72	483.60	3.0279	1.1023	0.081	484.55	1.1040	223.27	2.17	6.28
451.76	498.18	3.0133	1,1027	0.038	499.12	1.1047	229.97	2.17	6.28
465.28	513.14	3.0033	1,1028	0.009	514.07	1.1049	236.87	2.17	6.28
479.27	528.52	2,9966	1.1027	-0,009	529.41	1.1048	243.98	2.17	6.28
493.73	544.33	2,9924	1,1025	-0.022	545.18	1.1045	251.30	2.17	6.28
508.64	560.61	2.9900	1.1022	-0.029	561.41	1.1041	258.84	2.17	6.28

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Table 2.4. Behavior of the modified model in a growing economy

Y/P	Y	% ∆¥	P	₹∆₽	¥*	P*	М	V	R
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
216.34	216.34	0.0000	1.0000	0.000	216.34	1.0000	100.00	2.16	6.24
230.53	230.53	6.5602	1.0000	0.000	216.34	1.0000	110.00	2.10	5.32
222.85	234.47	1.7088	1.0521	5.214	232.00	1.0000	110.00	2.13	5.79
219.45	236.28	0.7725	1.0767	2,335	236.21	1.0575	110.00	2.15	6.02
217.39	237,39	0.4711	1.0920	1.419	238.04	1.0841	110.00	2.16	6.16
216.06	238,12	0.3076	1,1021	0.925	239.10	1.1003	110.00	2.16	6.26
215.21	238.59	0,1968	1.1086	0.591	239.74	1.1107	110.00	2.17	6.32
214.71	238.87	0.1179	1.1126	0.354	240.09	1.1171	110.00	2.17	6.36
214.44	239.02	0.0620	1.1146	0.186	240.26	1.1206	110.00	2.17	6.38
214.34	239,08	0.0232	1.1154	0.069	240.28	1.1221	110.00	2.17	6.39
214.36	239.07	-0.0031	1.1153	-0.009	240.22	1.1222	110.00	2.17	6.39
214.44	239.02	-0.0202	1.1146	-0.060	240.09	1.1214	110.00	2.17	6.38
214.57	238.95	-0.0305	1.1136	-0.091	239.94	1.1200	110.00	2.17	6.37
214.73	238.86	-0.0360	1.1124	-0.108	239.76	1.1183	110.00	2.17	6.36
214.89	238.77	-0.0382	1.1111	-0.114	239.59	1.1165	110.00	2.17	6.35
215.06	238.68	-0.0382	1.1099	-0.114	239.41	1.1147	110.00	2.17	6.33
215.21	238.59	-0.0366	1.1086	-0.109	239.25	1.1130	110.00	2.17	6.32
215.36	238.51	-0.0342	1.1075	-0.102	239.10	1.111.3	110.00	2.17	6.31
215.49	238.44	-0.0312	1.1065	-0.093	238.96	1.1098	110.00	2.17	6.30
215.62	238.37	-0.0281	1.1055	-0.084	238.83	1.1084	110.00	2.17	6.29
215.72	238.31	-0.0249	1.1047	-0.074	238.72	1.1072	110.00	2.17	6.28
215.82	238.26	-0.0219	1.1040	-0.065	238.62	1.1061	110.00	2.17	6.28
215,90	238.21	-0.0190	1.1033	-0.057	238.53	1.1052	110.00	2.17	6.27
215.97	238.17	-0.0164	1.1028	-0.049	238.45	1.1044	110.00	2.17	6.27
216.03	238.14	-0.0141	1.1023	-0.042	238.38	1.1037	110.00	2.16	6.26
216.08	238.11	-0.0120	1.1019	-0.035	238.33	1.1031	110.00	2.16	6.26

Table 2.5. Behavior of the modified model in a stationary economy

formulate monetary policy in the no growth version of the model and interpret the results as deviations from the steady growth rate time path. This is a significant conceptual simplification.

Second, the model provides a bridge between the two major disjoint branches of economic theory.

In long run equilibrium it displays all properties generally associated with the Monetarist position: Long run money neutrality with prices determined in the monetary sector through the quantity equation. Transient monetary disturbances or once and for all monetary changes have no effect on long run equilibrium in the real sector. Monetary growth at the same rate as exogenous demand and full employment real output results in an aggregate price level which is asymptotically constant through time, regardless of initial conditions. In the short run however it behaves as an Income--Expenditure model: Changes in the money stock affect the quantity of real investment and the rate of interest and through this mechanism lead to changes in real output.

Only within the context of such a model is it possible to attack the question of optimal monetary policy formulation in a realistic fashion. It is generally accepted that an increase in the money stock affects both real output and prices even in the short run. Within the context of this model one may compare explicitly the full implications of

particular policy proposals. Given an objective function which places appropriate weights on full employment and inflation control, it is possible to compute the best time path for the money stock. Without such a model one may only say that money should be a bit easier or a little bit tighter--but one cannot say how much.

CHAPTER III

Section A

The objectives of monetary stabilization policy are generally characterized as (1) full employment and (2) price stability. In principal it is possible to specify an objective function which is capable of measuring the extent to which these two objectives are met. Using such a function together with a model of the economy which incorporates both of the arguments of that function as endogenous variables, it is possible to compare the performance of various strategies of monetary control. Indeed, given the definition of comparability established in the objective function, it is possible to determine that particular monetary control strategy which is optimal. Such an exercise clearly is only possible within the context of a model in which both prices and income are endogenous.

The dynamic model developed in the preceding chapter provides the theoretical framework within which questions of optimal monetary policy formulation may be investigated. Within such a framework economists may be led to ask the proper questions. It is hoped that such questions may then lead to answers which are economically meaningful.

Economists of the Monetarist persuasion view control of the money stock by the central bank as a powerful and flexible

tool for economic stabilization policy. However the published literature contains relatively few discussions of what would indeed constitute an optimal policy. What little debate there has been has focused on the "rules versus discretion" controversy. The precise nature of the problem at issue has been unclear, however, in part because neither side has defined its case in terms of some specified, intertemporal optimality criterion and also because explicit models of the economy have not generally been employed as a theoretical basis for discussion. The present approach is a first step toward resolving this issue.

Proponents of discretionary action on the part of monetary authorities have argued, plausibly enough, that the central bank can substantially influence movements in the stock of money held by the private sector and hence, through the Quantity Theory mechanism, should be able to influence movements in nominal income. This position has been subjected to a three pringed attack. The most serious objection stems from the fact that central bank actions seem incapable of dominating movements in the money stock in the extreme short run, particularly monthly movements. Even in the case of quarterly averages, proximate determinants of the money stock not under central control, the currency to deposit ratio for example, play important and very independent roles (c.f., Cagan, 1965). Under this line of reasoning aggregates directly

controllable by the central bank--the monetary base, free reserves, interest rates, etc.--become the appropriate control variables (c.f., Pindyck and Roberts, 1974). The second line of attack stems from the inability of policy makers to estimate accurately future and even current values of important economic aggregates, especially private investment, fiscal receipts and expenditures, and national income. Finally, discretionary monetary policy is severely complicated by the lag in effect of monetary action. Since a change in monetary policy makes itself felt not only in the current time period but also for several future periods, appropriate policy determination at any time cannot be divorced from previous policy directions. This third difficulty further compounds the first two problems. It is interesting to note that in no case is the attack based on criteria of dynamic optimality of some alternate policy model.

Such issues are clearly of fundamental importance. However they are not the points at issue in the present discussion. It is assumed here that the money stock is in fact an exogenous policy instrument. It is further assumed that reliable estimates of exogenous components of aggregate demand are available to policy makers.

These are restrictive assumptions. However they allow this discussion to center on even more basic issues: the determination of an optimal monetary policy and the gains

which result from the pursuit of such a policy. If such gains can be clearly demonstrated, it then becomes fruitful to devise methods to control money more closely and to obtain improved economic forecasts.

We have reached the stage in the development of macroeconomic theory where it is generally recognized that the relationships being dealt with are essentially dynamic in nature. Furthermore the focus of the attention of the economic analyst has shifted. Not too many years ago economists concerned themselves with the "long run", with "normal" prices, with the "steady state equilibrium". More recently, however, macroeconomists have taken Keynes' maxim to heart: "In the long run we all are dead." Concern has shifted to "short run" analysis with emphasis on adjustment mechanisms and policy formulation.

Beginning at least with John Maynard Keynes, the principal expository tool of macro analysis has been that of comparative statics. Comparative static analysis is most properly suitable to the comparison of alternate steady state solutions each of which might be expected to prevail for such a long time duration that any intermediate adjustment periods may be safely ignored. The thrust of analysis is aimed at the properties of the steady state rather than at the properties of the system during the adjustment period. This comparative static mentality leads to a decision making

process characterized by the comparison of various policy proposals and their associated long run results. Some consideration may be given of course to the expected behavior of the economic system during the adjustment period. Such discussions however are not generally of central importance, and the models employed often are poorly adapted to such analysis.

This difficulty becomes further compounded when the properties of the dynamic adjustment path are considered equally as important as the steady state properties of the system. Dynamic systems, particularly nonlinear systems are extremely complex. Well intentioned policies formulated with an incomplete appreciation of the dynamic relationships being dealt with may well lead to unexpected and even perverse results. Additionally, even in the simplest of dynamic systems the number of possible policy proposals multiplies very quickly. As a practical matter it is generally impossible to consider all possible proposals; often debate centers solely on a comparison of a very few extreme positions.

Policy formulation by such a primitive technique may be characterized as "best" or "optimal" only in a very restricted sense. One may reasonably suspect that the policy which is indeed optimal has gone unnoticed due to the complexities inherent in any dynamic system. This seems to have been the case in the debate on the wisdom and proper form of an optimal

monetary stabilization policy. Techniques are needed which are capable of isolating that time path of the policy variables which is truly optimal in a dynamic setting.

Dynamic problems of this sort fall within the mathematical realm loosely referred to as control theory. While control theory traces its roots to the classical variational calculus, the modern approach to such problems derives more directly from the work of Pontryagin, Bellman, and many others. Analytic solutions are obtainable for only the simplest classes of control problems. However these modern approaches suggest numerical techniques which are capable of generating solutions for a much broader spectrum of problems.

The control problem with which we are dealing here consists of three basic elements. First a mathematical description of the system to be controlled has been developed in Chapter II. Second a performance index must be constructed by which the behavior of the system may be evaluated. Finally one must specify the set of admissible control imputs, the money stock in the present case, and the set of uncontrollable imputs, in this case the initial conditions of the state variables, exogenous demand, and the level of full employment GNP.

There are innumerable indices which could be created to measure the efficacy of stabilization policy for any particular point in time. Plausibly the performance index

chosen should impose a penalty for excessive rates of unemployment, i.e. levels of real output below the full employment level. In addition a penalty will be levied for any price changes. The goal of price stability is more closely related to the minimization of period to period price changes than to maintenance of a constant level of prices through time. One such index is defined below as the function COST(t), a weighted sum of these two factors, with Z the relative weight to be placed on the inflation component.

$$COST(t) = \left(\frac{Y_t - Yf_t}{Yf_t}\right)^2 + z\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)^2 \quad (3.1)$$

Thus COST(t) is a function only of Y desired, a new variable could be introduced into the system making COST(t) a function only of the state of the system at t. Note further that new variables X1 and X2 could be defined such that

$$x1_{t} = \frac{Y_{t}}{Yf_{t}}$$
$$x2_{t} = \frac{P_{t}}{P_{t-1}}$$

COST(t) = $(X1_t - 1)^2 + z \cdot (X2_t - 1)^2$ (3.2)

Cost(t) is then a quadratic form in the new state variables. If the purpose of the present discussion were a proof of the existence of an optimal monetary path or with the computation

of the analytical solution for such a path, such a transformation would be desirable. There is a well developed body of literature dealing with optimal control of systems with quadratic cost functionals. The principal purpose here is expository however. It is desirable to remain as close as possible to the original model. The existence of an optimal solution will be assumed here. This assumption will later be buttressed by the computation of a solution which is, by all appearances, optimal.

The function COST assigns a scalar index number by which the performance of the economy may be evaluated at any point in time. In a consideration of the efficacy of monetary stabilization policy we wish to somehow capture the net performance of that policy over a horizon at least as long as a complete business cycle. Monetary action taken today affects the economy today, tomorrow, and on into the future.

It is not immediately clear what criteria should be used to judge whether policy A is superior to policy B. We might require, for example, that the following relationship hold between the cost functions $COST_A(t)$ and $COST_B(t)$ generated by A and B respectively.

> $COST_{A}(t) \leq COST_{B}(t), \quad t = 1, 2, ..., N$ $COST_{A}(t) < COST_{B}(t) \quad \text{for at least one t}$ (3.3)

An alternate, and somewhat weaker criterion, is that of requiring

$$\sum_{i=t}^{N} COST_{A}(i) < \sum_{i=t}^{N} COST_{B}(i)$$
(3.4)

This second alternative is the one employed in control theory. It allows the formulation of policies which permit the intertemporal trade off of current costs for future benefits. For example it may be desirable to temporarily create rather large levels of unemployment through a tight monetary policy in order to achieve the goal of price stability in the future.

Thus the objective of control in this model is that of minimizing

$$J(t) = \sum_{i=t}^{N} COST(i)$$
 (3.5)

subject to the equations governing the dynamics of the system, Equations Al to A4 of Appendix A, and the given initial conditions of the system.

There are various analytical methods designed to deal with the general control problems of the form

$$\begin{array}{cccc}
& N \\
\text{Min } J(t) = & \text{Min } \Sigma & \text{COST (i)} \\
& u & u & i=t
\end{array}$$

subject to the dynamic system

 $X_{t+1} = f(X_t, u_t, t)$

with

$$X_t$$
 a k x l vector;
 X_o : given initial conditions (3.6)

The approach pioneered in the work of Pontryagin and others (1962) involves the conversion of this constrained minimization problem into an unconstrained problem by the introduction of a vector of co-state variables serving a function analogous to that performed by the Lagrangean multipliers in a static minimization problem. The auxiliary problem then takes the form

$$\underset{u}{\text{Min}[H(x_t, \lambda_t, u_t, t) - \lambda_t x_{t+1}]}$$
(3.7)

where $H = J + \lambda_{+}f$.

For an optimal interior solution to this new problem it is necessary that this new function represent a stationary solution with respect to all the variables of the system (Athans and Falb, 1966). This requirement leads to the set of necessary first order conditions often called the canonical equations of the system. These equations correspond to the Euler Equations of the Calculus of Variations.

$$\frac{\partial H}{\partial u_{t}} = 0$$

$$x_{t+1} = \frac{\partial H}{\partial \lambda_{t}} \text{ for } t = 1, 2, \dots, N$$

$$\lambda_{t} = -\frac{\partial H}{\partial x_{t+1}}$$
(3.8)

For some special cases these equations may be solved analytically for the optimal control path U*, X*. Typically the first condition is used to eliminate U from the remaining equations. One is left then with the problem of solving a two point value problem for a system of 2k first order difference equations. If the number of state variables is very large or if the difference equations are nonlinear, this is not a trivial problem. The first order conditions 3.8 provide necessary conditions for an interior extremal solution. Second order properties of the system may be investigated to insure the extremal is indeed a minimum. The first expression in 3.8 requires the auxilary function to be an extremal with respect to the control vector. To guarantee that this occurs at a relative minimum the second partial derivative, the Hessian determinant in vector space, must be nonnegative, the classical Legendre condition.

$$\frac{\partial^2 H}{\partial u^2} \ge 0 \tag{3.9}$$

Further to guarantee global minimization the auxiliary function must be weakly concave in U, the Wierstrass condition

$$H(\mathbf{X}_{t}^{*}, \lambda_{t}^{*}, \mathbf{U}_{t}^{*}, t) \leq H(\mathbf{X}_{t}, \lambda_{t}, \mathbf{U}_{t}, t) + (\mathbf{U}_{t}^{*} - \mathbf{U}_{t}) \frac{\partial H}{\partial \mathbf{U}_{t}} (\mathbf{X}_{t}, \lambda_{t}, \mathbf{U}_{t}, t)$$
(3.10)

Here U*, X*, λ * represent optimal control paths, and U, X, λ represent any other admissible trajectory.

There are two basic classes of numerical solution techniques for the general control problem. Indirect methods are those which employ iterative techniques to approximate the solution of the 2k difference equations described in the preceding paragraph. There are standard iterative computational programs for solving certain types of systems of difference or differential equations. Additionally, the functional form of the solution is known for several standard types of control problems. For problems yielding canonical equations which conform to one of these standard types an indirect solution method may be employed. The most common example is the case in which the objective function in a quadratic form in both the state and control vectors and the dynamic equations are linear in the difference operator. Solutions may be obtained for systems containing a rather large number of state variables.

Direct solution methods are those in which the optimal control is determined by operating directly on the performance index J. An initial trial control sequence ul is selected and the system is simulated. Using information based on J, the gradient of J, and in some methods the second derivative, the initial control sequence is modified to produce a new trial sequence u2. Hopefully upon iteration the control sequence converges to the optimal sequence u* which minimizes J. Methods which employ only first order information fall into the generic class of steepest descent techniques (Curry, 1944). They generally display the property of fast convergence to a neighborhood about u*, slow convergence thereafter. Second order techniques converge more slowly initially, but more q quickly later on, and thus are sensitive to the initial trial solution choice, ul. Currently the most popular algorithms are the Conjugate-Gradient and Davidon methods; they employ both first and second order information combining the advantage of each.

An extensive discussion of solution algorithms may be found in Bellman and Dreyfus (1962) along with an annotated bibliography. An application of Conjugate-Gradient and Davidon methods to two sector economic growth models may be found in Keller (1972). This contains a Fortran code used in the solution of this type of problem as well as a theoretical discussion. An interesting discussion of the solution of discrete
time control problems by nonlinear maximization techniques may be found in Fair (1974).

Computational solution algorithms for control problems are extremely problem particular. The particular solution technique appropriate to a given problem is dependent on the complexity of the relationships involved, the amount of a priori information available, the degree of accuracy desired in the result, and the amount of money available to buy computer time. General programs designed to handle a wide range of nonlinear problems are computationally expensive; often however significant short cuts may be suggested by the nature of the problem under consideration. Thus the potential user is advised that he will probably end up writing his own program unless his problem is of the linear-quadratic type mentioned in the discussion of indirect solution techniques.

Section B

Direct solution techniques often rely heavily on the principle of the dynamic programming approach to the solution of sequential optimization problems developed and popularized by Bellman and Dreyfus (1957, 1962). The sequential solution algorithms employed have their basis in Bellman's Principle of Optimality (1957, p. 83).

An optimal policy has the property that, whatever the initial state and decision are, the remaining decision must constitute an optimal policy with regard to the state resulting from the first decision.

Aris (1964, p. 27) expressed the proposition somewhat more succinctly. "If you don't do the best with what you happen to have got, you'll never do the best you might have done with what you should have had."

Properly speaking, dynamic programming is not in itself either a direct or indirect solution method, but rather a fruitful approach to the solution of a wide variety of sequential problems. Consider the problem

$$\underset{u}{\text{Min } J(t) = \sum_{i=t}^{N} COST(x_{i}, u_{i}, i) \qquad (3.11)$$

The principle of optimality allows the problem to be broken into two parts. Thus the general control problem may also be formulated as a problem of sequential decision making.

$$J_{(t)}^{*} = \min_{u} COST(x_{t}^{*}, u_{t}^{*}, t) + \min_{u} J(t+1)$$
(3.12)

This fundamental recurrence relationship permits the iterative solution of the system by proceeding in reverse order from t = N to t = 1. Thus at each stage the second term on the RHS has already been minimized; one is left with the problem of the minimization of current costs given the state at t.

This approach may be combined with a direct method optimization search technique to arrive at an optimal solution to the problem. An initial trial control trajectory is selected arbitrarily and used to generate the time path for

the state variables x through time. Then, working back from the terminal period to the initial period, the initial control sequence is modified at each stage to move the cost function in the negative gradient direction. After a number of iterations the trial control path converges, hopefully, to the optimal path.

One method for computing the gradient direction for a function with only one control variable is to compare the value of the function at u_t with its values at $u_t + e$ and $u_t - e$ for some small, positive e. If $f(u_t + e)$ is less than $f(u_t)$ an increase in the trial control u_t is appropriate. If $f(u_t - e)$ is less than $f(u_t)$ a decrease in u_t is called for.

Due to the lag structure embodied in the present model one slight modification of this basic algorithm is called for. The state of a system is the vector of variables containing that minimum body of information available at t which summarizes a sufficient quantity of information about the past history of the system such that, together with information about the future path of the control variables and other exogenous system inputs, the future trajectory of the system may be computed. Thus for the present model a complete state description at time t is given by all endogenous variables indexed at t plus the price index and nominal income lagged one period. For example the boundary conditions of Appendix A provide a sufficient description of the state at t = 0.

Thus a marginal change in M, the control variable in the model, affects directly not only the state at t but also the state of the system in the next period. Since the dynamic iteration algorithm works in reverse order, an extension of the method is necessary to compensate for such lag effects. Further there are significant indirect effects on the state at t + 2 which result from a change in M. Consideration of these indirect effects, while not necessary, improve initial convergence properties without greatly increasing computation costs. It does so however at some sacrifice of convergence speed and sensitivity in the neighborhood of the optimal path.

The fundamental recursive relationship used here is a slight modification of that described earlier. At each stage only M is modified. However the criteria upon which this modification is based include consideration of direct and indirect lagged effects. The new term is assigned the code name TCOST.

$$TCOST(t) = \sum_{i=t}^{t+2} COST(i)$$
 (3.13)

The precise nature of the computational procedure is most easily summarized in a basic process flow diagram. Such a diagram follows as Figure 3.1. The computer code used to execute this algorithm may be found in Appendix D.

The algorithm displays good convergence properties. It is not particularly sensitive to the initial trial control

path, although the better the initial guess the faster is the convergence. Typically the algorithm converges after thirteen or fourteen iterations using about 15 seconds CPU time for the twenty period cases considered in the next chapter.

Conceptually the problem may be viewed as that of choosing N variables, M, through M_N , to minimize a nonlinear function of 2N variables, Y, through Y_N , and P, through P_N , subject to the constraints of N nonlinear relationships imposed by the dynamic structure in each period. Considering the complexity of the problem, the algorithm generates solutions with surprising speed. The method is a graphic illustration of the power of the dynamic programming approach to sequential problem solving.

Figure 3.1. Flow chart

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CHAPTER IV

Section A

The necessary tools have now been developed to compute an optimal monetary stabilization policy. Clearly the monetary policies arrived at are particular to the form of the objective function used to measure policy performance and to the nature of the dynamic system used to describe the economy. Computations were carried out using cost functions representative of a variety of policy goal orientations. Results of these exercises are the subject of this chapter.

This paper is primarily concerned with the issues raised by optimality analysis. The problem of the specification of the proper form for the objective function or the intricacies of the dynamic structure of the model are of only secondary interest here. The forms of the objective functions and the dynamic structure employed have been chosen as a necessary compromise between realism and solvability.

Problem A: A business cycle problem

In this and subsequent sections we wish to examine the economic stabilization question. A fluctuation in exogenous demand will be used to generate a "business cycle" of twenty periods duration. Recall the investment equation of the model, Equation A7 of Appendix A.

$$I_{t} = B_{t} \cdot R_{t}^{EI}$$
 (4.1)

where

$$B_{t} = b_{I} \cdot \prod_{i=1}^{t} \{(1 + RBI_{i})(1 + RYf)\}$$

The function

$$RBI_{i} = 0.06 \sin \left(\frac{2i\pi}{N}\right)$$
 (4.2)

is used to generate a sine wave business cycle. If interest rates were to remain constant, this would generate a cyclic variation in real output and investment. Simulations were undertaken in a stationary economy with RYf set equal to zero.

To provide a benchmark for use in comparing alternate policies, the model was simulated with a procyclic monetary policy. Next a Friedman Rule policy was generated. Finally, using this path as an initial trial estimate, the dynamic programming algorithm was employed to yield an optimal time path.

Optimal controls were first generated for objective functions of the form

$$J(t) = \sum_{i=t}^{N} \left[\left(\frac{Y_{i} - Y_{f}}{Y_{f}} \right)^{2} + Z \cdot \left(\frac{P_{i} - P_{i-1}}{P_{i-1}} \right)^{2} \right]$$
(4.3)

where the parameter Z reflects the relative policy importance of the price stability objective versus the full employment objective. Z weights of 0.1, 1.0, and 10.0 were used to allow

comparison of policies employing normal full employment, balanced full employment and price stability, and price stability policy orientations.

The first case considered was that in which Z = 1. The procyclic policy yielded a total cost J = 240.30 points. As would be expected, the Friedman Rule policy performed considerably better with a total cost of J = 26.97 points. After five iterations by the dynamic program this was reduced to J = 6.33. By the end of nine more iterations J was reduced to J = 0.64. At this point convergence within the tolerances of the program had been reached; further iterations produced no further modification of the control variable. The program could be modified to achieve closer convergence by reducing the stepsize variable and increasing the sensitivity of the numerical gradient approximation technique.

The fact that the cost functional converges essentially to zero is extremely interesting. It suggests that monetary policy goals of maintaining both normal full employment and price stability are fully achievable over the course of a business cycle. Thus any policy which maintains real output at the full employment level will not lead to inflation. Conversely a zero price change policy can only be effected by maintaining full employment. This proposition is a logical result of the structure of the model. Such a zero cost control path will be referred to as the Golden Rule path.

Since the Golden Rule trajectory is fully obtainable in this case, it follows that this path is independent of the relative weights placed on the full employment and price stability components of the objective function. Given a set of initial conditions which lie on the path, the price stability and full employment goals are equivalent.

Moreover confirmation of this proposition under various policy weight ratios was used as a test of the accuracy of the dynamic programming algorithm. Calculations for Z = 0.1, Z = 1, and Z = 10, where Z is the relative weight placed on the price stability term of the objective function, yielded monetary trajectories which differed at most by one percent from the average at any point in time.

The results of this exercise are most easily presented in graphical form since qualitative comparisons rather than the actual data points are of primary interest. While the graphs are largely self-explanatory, several points are of particular interest.

In Figure 4.1 the optimal monetary policy is, as one would expect, counter cyclical. It is interesting to note however that monetary movements are apparently in phase with the exogenous demand cycle--even though monetary changes affect the state variables directly for two periods with strong indirect effects for one more period. Finally the horizon effect usually associated with finite time free

endpoint problems does not appear since the value of the cost function can be driven to zero in every period. No trade-off occurs between the full employment output and price stability objective.

Normal income and prices, and hence real income, are completely stabilized under this policy objective. The compatability of the two policy goals is not surprising given the nature of the dynamic structure of the model. Nonetheless discovery of the precise monetary policy which meets both goals at once is of some considerable interest.

Finally in Figure 4.7 and 4.8 it is interesting to observe that velocity and interest rates behave in a procyclic fashion. This is consistent with empirical observations about the behavior of these variables. More surprising however is close agreement of the magnitude of these two variables among the various policies. There is little difference in the maximum rate of interest produced by the procyclic and optimal counter cyclic policies. In Section C of this chapter it is shown that interest rates can be stablized. This is not accomplished however by conducting a counter cyclic monetary policy.

It is tempting to offer a few separate comments on the behavior of the system under the procyclic policy. The behavior over the first ten periods is of particular interest in light of the current U.S. economic situation. A case could

be made that the Federal Reserve has pursued a procyclic policy from 1968 through 1973 during a period when exogenous demand rose and then fell relative to trend. In the present model such a procyclic policy produced an in-phase movement in nominal income (Figure 4.2). However, due to high levels of inflation generated in the process real income began to decline after the third period and falls below the full employment level after the fifth period (Figures 4.3 and 4.4). However the inflation rate does not fall to zero until the eighth period, and interest rates remain above normal until the twelfth period (Figure 4.8).

Easy money, which in a static IS-LM model generates low interest rates has precisely the opposite effect in a dynamic setting. During a business cycle boom the interest rate rises under all three monetary policies to roughly the same level. It would be tempting for the central bank to attempt to ease the credit crunch at the peak of the boom by an easy money policy. This policy will not necessarily achieve the desired result. Such a policy merely generates higher aggregate demand, a high rate of inflation which soon dominates income changes and crowds out real output and employment, and finally higher rates of interest for an extended period.

Figure 4.1. Problem A: A stabilization problem

Ordinate:		Money stock
Abscissa:		Time index
Key:	O: ∆: +:	Procyclic policy Friedman Rule Optimal control

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Figure 4.2. Problem A: A stabilization problem

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Ordinate:Nominal incomeAbscissa:Time indexKey:O:Procyclic policy Δ :Friedman Rule+:Optimal control

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Figure 4.3. Problem A: A stabilization problem

Ordinate: Price index Abscissa: Time index

Key: O : Procyclic policy ∆ : Friedman Rule

+ : Optimal control

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Figure 4.4. Problem A: A stabilization problem

Ordinate: Real income Abscissa: Time index Key: O : Procyclic policy ∆ : Friedman Rule + : Optimal control



Figure 4.5. Problem A: A stabilization problem

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Ordinate: Inflation rate Abscissa: Time index

Key: O : Procyclic policy ∆ : Friedman Rule

+ : Optimal control

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Figure 4.6. Problem A: A stabilization problem

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Ordinate: Percent change nominal income Abscissa: Time index



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Figure 4.7. Problem A: A stabilization problem

Ordinate: Income velocity of money Abscissa: Time index

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Key: O : Procyclic policy Δ : Friedman Rule

+ : Optimal control



Figure 4.8. Problem A: A stabilization problem

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Ordinate: Interest rate Abscissa: Time index Key: O : Procyclic policy Δ : Friedman Rule + : Optimal control

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Section B

In the problem of Section A if an optimal monetary policy is pursued throughout the business cycle, the cost function J is minimized at J = 0. There exists a monetary policy which maintains real output at the normal full employment level and, equivalently for this model, leads to a stable level of prices. For want of a better name we will refer to this full employment, zero inflation path as the Golden Rule stabilization path. In Section A initial conditions were chosen which placed the economy on the Golden Rule path. It is logical to consider next the optimal monetary policy when the initial conditions do not place the economy on this Golden Rule path.

Two distinct but related issues are of interest here. First is the optimization issue itself: which control trajectory is indeed optimal given the initial conditions policy makers have to work with? The second is the question of the stability of the Golden Rule path: is the Golden Rule path a knife edged equilibrium which can only be maintained from a particular starting point, or does it represent a turnpike toward which all optimal trajectories converge given a long enough horizon?

Problem B: A policy shift problem

Policy makers, being human, make mistakes. Policy objectives change from time to time; certainly not all

objectives, if pursued optimally, are consistent with the Golden Rule path described above. Assume that, for whatever reason, a procyclic monetary policy is pursued for five time periods. At that point policy emphasis changes; it is desired from then on to minimize the cost function of Section A. Thus the optimization problem begins at period t + 6 with initial conditions such that real output lies below the full employment level with prices above the equilibrium level and rising. Furthermore the economy is headed into the downturn of the business cycle.

For reference, if a procyclic policy were pursued from this point on, the total value of the objective function over the remaining 15 periods would be J = 159.44. A procyclic policy should be easy to improve on. Recall that in the present model long term trend factors have been divided out to allow easier consideration of the cyclic stabilization problem. The steady rate of monetary growth Friedman Rule thus translates into a constant money stock rule. This rule yields a total cost of J = 50.80, a considerable improvement over the procyclic policy. Next the optimal trajectory was computed. The program converged after 13 iterations with a computed cost over the 15 period horizon of J = 12.71.

Results of this exercise are summarized in Figures 4.9 through 4.13. Note the substantial difference between the Friedman Rule trajectory and the optimal money stock

trajectory in Figure 4.9. Maximum deviation is on the order of 10%; in the Golden Rule initial conditions example of Section A the maximum deviation was 2%. Optimal policy is clearly extremely sensitive to the state of the economy which obtains at the time such policy is determined.

Throughout the planning horizon prices are increasing (see Figure 4.11). Yet optimal policy results in extremely rapid monetary expansion over much of the horizon. At first glance this is paradoxical. One may gain some intuitive feel for why the optimal monetary time path behaves the way it does by reference to Figures 4.12 and 4.13. Real income at the start of the planning horizon t = 6 is substantially below the normal full employment level. Y/P is 2 3.97 as opposed to Yf/P of 216.34. In addition exogenous demand declines from t = 6 to t = 15. The inflation control objective is being sacrificed to avoid excessive unemployment.

The objective function used in the present example placed equal weight on the full employment and price stability objectives: Z = 1.0. Note in Figure 4.12 that real income is gradually increasing in the direction of the full employment level 216.34. Throughout the same period the rate of inflation gradually falls toward zero, Figure 4.13. A more restrictive monetary policy in the initial periods would decrease the rate of inflation more quickly at a cost of lower levels of real output.

In Section A with initial conditions which lay on the Golden Rule path optimal control resulted in a zero cost trajectory. Furthermore that trajectory is independent of the relative weights placed on the full employment and price stability components of the objective function.

For initial conditions which do not lie on the Golden Rule path, however, the optimal trajectory is not policy weight independent. Thus an objective function which places greater relative weight on the full employment objective will cause this objective to dominate in cases where the components of the objective function conflict. Such conflicts arise only when the system has been allowed to drift away from the Golden Rule path.

This example provides a graphic illustration of the turnpike properties of the system. At time t = 6 when the optimization procedure is begun the system is well away from the Golden Rule path. By the end of the planning horizon however the system is very close to the Golden Rule path. Single period objective function costs have declined from 2.54 in period 6 to 0.38 in the final period. Since in each period any deviation from the Golden Rule path generates positive costs, this is an indication that the trajectory is approaching the zero cost path.

Figure 4.9. Problem B: A policy change problem

Ordinate: Money stock Abscissa: Time index

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Key: O : Friedman Rule Δ : Optimal control



Figure 4.10. Problem B: A policy change problem

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Ordinate: Nominal income Abscissa: Time index Key: O : Friedman Rule Δ : Optimal control


Figure 4.11. Problem B: A policy change problem

Ordin	ate:	Price index
Absci	.ssa:	Time index
Key:	o : ∆ :	Friedman Rule Optimal control



Figure 4.12. Problem B: A policy change problem

Orđin	ate:	Real	income
Absci	ssa:	Time	index
Key:	0:	Fried	lman Rule

 Δ : Optimal control

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Figure 4.13. Problem B: A policy change problem

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Ordinate: Inflation rate Abscissa: Time index

Key: O : Friedman Rule Δ : Optimal control

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An optimal policy has been computed over the planning horizon. This is not, however, a zero cost policy. It is a cost minimization policy. There is, in short, a cost associated with any set of initial conditions not on the Golden Rule path. This cost cannot be avoided. In a sense policy makers must pay for past mistakes. Furthermore, use of fiscal policy instruments in addition to monetary control instruments would in no way affect the costs of returning the system to a stable price, full employment trajectory. The optimal level of real output used to influence the price determination mechanism can be completely achieved by monetary policy alone. With this objective function fiscal tools are redundant.

An optimal trajectory may be computed for any set of initial conditions. Moreover, this trajectory will approach the Golden Rule path given enough time.

Section C

Full employment and price stability clearly are not the only possible stabilization policy goals. Among the many conceivable alternatives, the one mentioned most frequently is that of interest rate stabilization. In the U.S. the precise nature of interest rate stabilization objectives has varied from time to time. Immediately following World War II it took the form of maintenance of a target rate of interest.

The interest rate goal currently pursued by the Fed is aimed more at minimizing period to period fluctuations -- or at least placing boundaries on the acceptable magnitudes of such fluctuations.

This section contains the results of simulations based on interest rate stabilization objectives. The results are not surprising. They do however point out the power and flexibility of the dynamic programming approach to policy formulation.

Problem Cl: An interest rate peg problem

The first policy objective considered was the policy of attempting to maintain some target rate of interest, R*. The objective function used was

$$J(t) = \sum_{i=t}^{N} (R_{i} - R^{*})^{2}$$
(4.4)

It is clear in the standard IS-LM model with an exogenous price level that this objective can be met by the generation of an appropriate procyclic monetary time path. The trajectory of course would not be a full employment trajectory.

In the present model it is not intuitively obvious that this objective can even be met. Levels of real output which differ from the full employment level generate price changes. The feedback relationships are relatively complex. Other things equal an increase in money drives down interest rates. It also drives up prices. But inflation under certain conditions implies an increase in nominal income relative to money with a resulting increase in interest rates. It is possible, a priori, that this will result in a catch up policy with money chasing interest rates during upswings and downturns in business activity and catching up only at turning points. It may not be possible to develop a zero cost monetary policy for this objective function.

For the parameter values used in this simulation, however, the system is apparently controllable for this objective function (see Figure 4.14). The initial trial trajectory used was the Friedman Rule trajectory. This time path is plotted along with the optimal solution as a benchmark. The complexity of the control relationships is reflected in the large number of iterations required to produce a solution. The value of the objective function for the initial trial was J = 8.64. After fifteen iterations this was reduced to J = 1.57, fifteen more iterations gave J = 0.86. The algorithm had not converged at this point. Changes were still being made in the values of M for the final few horizon periods. However the objective function was decreased regularly with each pass and appeared to be converging to some value in the neighborhood of zero. Further expenditures were judged to be unwarrented.

Attempts to hold down interest rates permit rapid increases in real output in initial periods. This in turn leads to a high rate of inflation. Ultimately it is inflation

Figure 4.14. Problem Cl: An interest rate peg problem

Ordinate: Interest rate Abscissa: Time index

Key: O : Friedman Rule Δ : Optimal control



rather than a decrease in the money stock which acts to lower real income (see Figures 4.15 and 4.16). The optimal monetary trajectory is found in Figure 4.17.

Such an interest rate objective clearly leads in this model to economic implications which are less than desirable. Such results are similar however to those observed by those central banks which have sought in the past to peg the rate of interest. Interest rate pegging is no longer viewed as a viable policy goal since it results in a loss of control over the principal objectives of monetary stabilization policy.

Problem C2: A mixed interest stabilization policy

An interest rate rule more in line with current practice is that of minimizing period to period fluctuations while at the same time pursuing the other goals of policy. This policy is aimed at promoting stability and regularity in financial markets. It has the further advantage of cushioning the impact of monetary policy on those sectors of the economy which are affected most rapidly by monetary actions, for example, housing construction.

A "pure" stabilization objective was tried initially.

$$J(t) = \sum_{i=t}^{N} (R_{i} - R_{i-1})^{2}$$
(4.5)

The solution to this problem is considerably more complex than that of the previous problem, a simple tracking problem. If a zero cost optimal control is obtainable for problem Cl, it is Figure 4.15. Problem Cl: An interest rate peg problem

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Ordinate:	Infaltion rate
Abscissa:	Time index

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Key O : Friedman Rule Δ : Optimal control



Figure 4.16. Problem Cl: An interest rate peg problem

Ordinate:	Real	income
Abscissa:	Time	index

Key: O : Friedman rule Δ : Optimal control



Figure 4.17. Problem Cl: An interest rate peg problem

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Ordinate:	Money stock	:k
Abscissa:	Time index	2

Key: O : Friedman Rule Δ : Optimal control



also optimal for this objective under the proper initial conditions. However for problem Cl the value of the target interest rate for each period is known from the start. Here the value of the target at each time t, R_{t-1} , varies with each successive iteration.

As might be expected, the solution algorithm did not display entirely satisfactory convergence properties under this objective function. Initial iterations quickly established a monetary control path qualitatively equivalent to the optimal path from problem Cl, reducing the value of the objective function to about 10% of the Friedman Rule cost. With successive iterations however the algorithm refused to converge beyond this level. Rather the monetary trajectory continued to wander about in the same general neighborhood for the next 80 iterations with no appreciable decrease in total cost. There are, apparently, a large number of monetary trajectories consistent with a "low" value for this objective function. The algorithm seems incapable of distinguishing among them. With minor adjustments the algorithm could no doubt be improved upon. Present interest however is centered on qualitative behavior.

Interest rate stability, if it is the sole objective of monetary policy, is thus qualitatively similar to an interest rate peg objective within the context of this model. Typically however interest stability is viewed as but one of several

policy objectives.

One possible objective function of this form is

$$J(t) = \sum_{i=t}^{N} \left[\left(\frac{Y_{i} - Yf}{Yf} \right)^{2} + \left(\frac{P_{i} - P_{i-1}}{P_{i-1}} \right)^{2} \right] + w(R_{i} - R_{i-1})^{2}$$
(4.5)

Figures 4.18 through 4.21 contain various plots of the control results under this objective function for interest stability weights, w, of 1.0, 10.0, and 100.0. The interest stability objective is incompatible with the goals of full employment and price stability. Further for trajectories away from the full employment path the full employment and price stability objectives are also incompatible. For w = 1.0 the solution trajectory is virtually identical to the Golden Rule path of Section A. However for higher weights the solution begins to take on qualitative characteristics similar to the pure stabilization and interest peg objectives discussed earlier.

If the primary objectives of monetary policy are full employment and price stability, great care must then be taken when formulating interest stability policy. It is fairly clear that persuit of an interest stability objective from period to period implies a loss of control over prices and employment equivalent to that experienced under an interest rate peg. Such considerations apply with particular force to Figure 4.18. Problem C2: A mixed interest stability objective

Ordinate: Interest rate Abscissa: Time index Key: O : w = 1 Δ : w = 10+ : w = 100



Figure 4.19. Problem C2: A mixed interest stability objective

Ordin Absci	ate: .ssa:	Money stock Time index
Key:	0 : ∆ :	w = 1 $w = 10$
	+ :	w = 100



Figure 4.20. Problem C2: A mixed interest stability objective

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Ordinate: Price index Abscissa: Time index Key: O : w = 1 Δ : w = 10+ : w = 100

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Figure 4.21. Problem C2: A mixed interest stability objective

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Ordin	ate:	Re al income
Absci	.ssa:	Time index
Key:	0 : ∆ : + :	w = 1 w = 10 w = 100



the so-called "defensive" operations of the Federal Reserve. The line between interest stabilization as discussed above and short term, "defensive" stabilization of financial markets may be extremely fine and indistinct.

CHAPTER V

The macro economic model developed in Chapter II and employed throughout the remainder of the paper is interesting in and of itself as a purely theoretical proposition. It provides a linkage between the two apparently disjoint branches of macro economics: short run Keynesian Income-Expenditure analysis, on the one hand, and long run Monetary Theory, on the other. It possesses a number of properties, both long run and short run, which make it a useful vehicle for demonstrating the basic outlines of macro economic relationships.

Nonetheless, no model, no matter how useful for exegetical purposes, can be convincing unless its basic relationships can be made plausible by empirical investigation. For the model in question most of the important relationships have been well documented. Short run properties of Keynesian models and long run properties of Quantity Theory models need no further verification for present purposes. The unique feature of this model is the inclusion of a price adjustment sector. This is the sector which drives the model. This is the equation which needs some empirical roots.

It has been postulated that there are three factors which combine to determine the price level at any point in time: (1) expectations, or more generally the immediate history of prices, (2) the relationship between actual and expected

levels of nominal income; and (3) the relationship between the actual level of income and what may be called the normal full employment level. It is this third relationship which is critical.

Much of the extant econometric work on the determinants of price changes may be interpreted in a manner generally consistent with the price determination thesis used here. The Phillips type studies emphasize the role of the unemployment rate, the extent to which the economy is operating at full capacity. Cost push studies generally emphasize one mechanism through which past price behavior influences current price behavior. These are the two critical linkages in the present model. These two types of price studies provide a firm empirical foundation for the general form of the price determination mechanism employed here.

One important element which has received little attention is the question of timing. Both unemployment rates and the history of price changes apparently are correlated with price changes. Which of these relationships dominates short term, say quarterly or monthly movements, and which is more long term in effect? Further what role is played by the postulated relationship between prices and actual versus expected levels of nominal income? These are the questions which will be investigated here.

Phillips studied the relationship between the rate of change of money wages and the level of unemployment. Building on this study, Lipsey observed a close correlation between the rate of change of money wages and the rate of price inflation as well as the level of unemployment. Friedman has suggested that the level of unemployment influences real wages, the movement of the money wage vis a vis the average level of prices. Most writers for the popular financial press as well as many cost-push theorists believe wage increases cause inflation.

One suspects, ex ante, that there is some truth in each of these positions. An example exists to support each of these positions; similarly, counter examples abound.

In this attempt to investigate the way in which short term changes in nominal income are divided between changes in real output and changes in the price level it is necessary to investigate the relationship between wages, prices, and employment. It is hoped that some extensions to the Phillips approach may be suggested.

It is of interest to begin with the relationship investigated by Lipsey. Recall the Lipsey results obtained with annual British economy data expressed in first central differences:

$$\frac{W'}{W} = 0.47 + 0.43 \left(\frac{1}{U}\right) + \frac{11.18}{(6.00)} \left(\frac{1}{U^4}\right) + \frac{0.038}{(.012)} \left(\frac{U'}{U}\right) + 0.69 \left(\frac{P'}{P}\right) (5.1)$$

A similar equation was fitted to quarterly U.S. data expressed in percentage rates of change with the following results.

$$\frac{W'}{W} = \frac{1.31^{*}}{(0.44)} - \frac{4.49^{*}}{(2.66)} \left(\frac{1}{U}\right) + \frac{81.70^{*}}{(35.78)} \left(\frac{1}{U^{4}}\right) - \frac{0.027^{*}}{(.0058)} \left(\frac{U'}{U}\right) + \frac{0.69}{(0.12)} \left(\frac{P'}{P}\right)$$
(5.2)
d.w. = 2.33 $S_{c}^{2} = 0.21$ 73 degrees of freedom

Inclusion of lagged independent variables in various combinations did not lower the mean square error and t tests on all lagged coefficients were uniformly nonsignificant. Estimation of this equation using the percentage change in weekly gross pay, w, instead of hourly wages gave similar results.

$$\frac{W'}{W} = 3.15^{*} - 14.24^{*} \left(\frac{1}{U}\right) + 157.71^{*} \left(\frac{1}{U^{4}}\right) - 0.072 \left(\frac{U'}{U}\right) + 0.78 \left(\frac{P'}{P}\right) (5.3)$$

d.w. = 2.53 $s_{e}^{2} = 1.65$ 73 degrees of freedom

From a theoretical standpoint these results are not appealing. In most Phillips-type studies the rate of price inflation is not introduced as a dependent variable. The usual inverse relationship between wages and unemployment then appears. Inclusion of this variable significantly alters the unemployment coefficients, however. The partial derivative of the equation with respect to the unemployment rate is slightly positive over normal ranges of U. However the estimated coefficient for the inflation variable is most interesting. Clearly there exists a strong positive relationship between wage and price changes.

Adherents to cost-push theories of inflation might suggest that the roles played by wages and prices should be reversed in a regression such as this. This yields

$$\frac{P'}{P} = -1.44* + 9.82 \left(\frac{1}{U}\right) - 135.9* \left(\frac{1}{U^4}\right) + 0.014* \left(\frac{U'}{U}\right) + 0.41* \left(\frac{W'}{W}\right) (5.4)$$

d.w. = 1.24 $S_e^2 = 0.13$ 73 degrees of freedom

The low Durbin Watson statistic suggests the presence of first order auto correlation among the error terms. Estimation of the error structure by fitting the estimated vector of errors, e_t , successively on e_{t-1} ; e_{t-2} , e_{t-3} ; and so on..., suggests the presence of second order autocorrelated errors $e_t = \rho_1 e_{t-1}$ + $\rho_2 e_{t-2}$ + v_t . To obtain unbiased estimators a second order correction for autocorrelation must be performed on the data. A simpler procedure is to insert two lagged dependent variables into the RHS of the equation. The resulting estimators are biased in finite samples. However if the error terms from the resulting system are uncorrelated, the bias is on the order of 1/n where n is the sample size (see Johnston, 1972, p. 304). For present purposes such bias falls within acceptable limits. The procedure offers the advantage of computational simplicity and a certain amount of intuitive appeal from the standpoint of economic theory. There is some

evidence to suggest that past increases in factor prices may be passed on to final consumers with some time lag and further that price expectations may play a major role in the process of price determination. To the extent that the lagged variables capture these tendencies explicitly their inclusion is defensible. Estimation by ordinary least squares gave the following results:

$$\frac{P'}{P} t = -1.08^{*} + 0.22^{*} \left(\frac{P'}{P}t-1\right) + 0.18^{*} \left(\frac{P'}{P}t-2\right) + 5.66^{*} \left(\frac{1}{U}t\right)$$

$$- \frac{89.25^{*}}{(22.71)} \left(\frac{1}{U_{t}^{4}}\right) + 0.013^{*} \left(\frac{U'}{U}t\right) + 0.27^{*} \left(\frac{W'}{W}t\right) + 0.15^{*} \left(\frac{W'}{W}t-1\right)$$

$$+ \frac{0.09}{(.063)} \left(\frac{W'}{W}t-2\right)$$

$$d.w. = 1.93 \qquad S_{e}^{2} \qquad 73 \text{ degrees of freedom}$$

$$(5.5)$$

Using weekly pay, w, as a proxy for wages yielded similar results.

$$\frac{P}{P} t = -0.94 t + 0.38 t \left(\frac{P}{P} t-1\right) + 0.28 t \left(\frac{P}{P} t-2\right) + 5.74 t \left(\frac{1}{U_{t}}\right) \\ - 72.95 t \left(\frac{1}{U_{t}^{4}}\right) + 0.012 \left(\frac{U}{U} t\right) + 0.082 t \left(\frac{W}{W} t\right) + 0.063 t \left(\frac{W}{W} t-1\right) \\ (0.029) \left(\frac{W}{W} t-2\right)$$

$$+ 0.024 \left(\frac{W}{W} t-2\right)$$
(5.6)

÷. ,

d.w. = 2.08 $S_e^2 = 0.095$ 73 degrees of freedom
These results tend to lend moderate support to the cost-push thesis. Note here that the partial derivative with respect to unemployment is negative over normal unemployment ranges, the expected sign. Increases in the rate of wage inflation may be expected to lead on the average to a slight increase in the rate of price inflation ceteris paribus. The sum of the wage rate coefficients however is only .5 in Equation 5.5 a permanent one percent increase in wage inflation should yield a one half percent increase in price inflation. The impact using weekly pay rather than the hourly wage is considerably weaker.

Equations of this type fit the data reasonably well. It is comforting to note the close relationship between wage and price changes. The relationship between the level of unemployment and the inflation rate in either form of the equation is of considerable theoretical interest.

The role played by the unemployment rate is somewhat ambiguous. In equations of the type 5.2 and 5.3 one is tempted to argue along with Lipsey that it is an index of excess supply in the labor market. In equations of the second type, 5.4 and 5.5, it is apparently acting an an index of general economic activity - prices on average have tended to rise faster during booms than during recessions. Neither of these explanations is entirely satisfactory however. Furthermore the unemployment rate has a relatively small impact on

either wage or price inflation. For example summing the intercept and the terms containing U from Equation 5.5 yields 0.01 for a 4% unemployment rate; -0.21 for a 6% rate. The average rate of price inflation is on the order of 0.72% per quarter. The unemployment rate is an important explanatory variable certainly; however it does not dominate the relationship.

Friedman (1968) has suggested in essence that this type of model is mis-specified, that the appropriate dependent variable is the rate of change of real wages and the proper independent variables are measures of excess supply in the labor market. Attempts to estimate such a relationship for quarterly data were not particularly successful. For example

$$\frac{W'}{W} - \frac{P}{P} = -0.52 + 3.27 \begin{pmatrix} \frac{1}{U} \\ 4.06 \end{pmatrix} - 5.52 \begin{pmatrix} \frac{1}{U} \\ 55.35 \end{pmatrix} \begin{pmatrix} \frac{1}{U} \\ \frac{1}{U} \end{pmatrix} - 0.0048 \begin{pmatrix} \frac{U}{U} \\ 0 \end{pmatrix}$$
(5.7)

No coefficients are significantly different from zero. Again introduction of lagged independent variables was of no help. Short term movements in real wages do not seem to be closely related to the level of unemployment over the sample period. These results are not surprising in light of the observed minor impact of the unemployment rate in preceding regressions. It seems quite reasonable that other market and institutional forces dominate such movements in real wages or that various random elements obscured the expected results.

There is a fundamental methodological difficulty with the approach taken thus far. The GNP Deflator is an index of final commodity prices; the wage rate series is an index of the price of a particular factor imput. In a Neoclassical world we would expect output prices and factor prices to change in a more or less parallel fashion during a general inflation. Wages and prices should move together although the movements need not be proportional. High multiple correlations found in Equations 5.2 through 5.6 may largely be the result of spurious correlation. The point is that in the search for a theory of general inflation it is dangerous to rely too heavily on the observed correlation between wages and prices. In a sense the wage rate is just another price; perhaps there exists some force in the economy which causes all prices to change.

In the search for such a causal factor it is tempting to assume the existence of a rather direct relationship between monetary movements and price changes. Such an impulse is well founded in the writings of all monetary theorists and has become a part of the conventional wisdom of economics. Indeed the thesis is well documented for cases of protracted hyper inflation (Cagan, 1956; Lerner, 1956; Patinkin, 1959). However hyperinflations are extraordinary phenomena, differing in many respects, one suspects, from the recent U.S. experience of more modest rates of price increase.

The thesis certainly cannot be substantiated by quarterly post war U.S. data. Regression by O.L.S. of the percentage rate of change of the Implicit Price Deflator on lagged price changes, unemployment, and the rate of change of money, narrowly defined, yielded the following results.

$$\frac{P}{P} t = -0.52 + 0.25 * \left(\frac{P}{P} t-1\right) + 0.26 * \left(\frac{P}{P} t-2\right) + 5.76 * * \left(\frac{1}{U_{t}}\right) \\ - 6.60 \\ (9.77) \left(\frac{1}{U_{t}^{2}}\right) + 0.033 \\ (0.074) \left(\frac{M1}{M1} t\right) + 0.005 \\ (0.094) \left(\frac{M1}{M1} t-1\right) \\ + 0.047 \\ (0.079) \left(\frac{M1}{M1} t-2\right) \\ d.w. = 2.03 \qquad S_{e}^{2} = 0.073 \qquad 73 \text{ degrees of freedom}$$
(5.8)

Regressions were also carried out using first central differences rather than percentage rates, using various lag and functional variations for the unemployment variable, and using a broader definition of the money stock, M2, rather than M1. Qualitative results were uniform in all cases: strong positive correlations were observed with past rates of price change, weaker inverse correlations with the unemployment rate, and virtually no observable correlation with the monetary variable.

The empirical evidence clearly does not lend support to the thesis that short term price movements can be appreciably influenced by monetary policy.

This does not suggest of course that money has no influence on the process of price level determination, but rather that the causal chain is somewhat more indirect and more complicated.

The usual interpretation drawn from studies of the Phillips variety is that there exists a trade-off between low levels of wage inflation or price inflation on the one hand and low levels of unemployment on the other. Such an interpretation is contradicted by periods of high employment and stable prices and by periods of high inflation and relatively high rates of unemployment, stagflation.

The usual interpretation is incomplete. Consider the following proposition. The Phillips relationship should be plotted in three dimensions, not two: the rate of current inflation, the rate of unemployment, and the rate of inflation in the previous period. There exists for example an apparent trade-off between wage inflation and unemployment ceteris paribus - i.e. at any particular level of expected price inflation. But the range over which this mechanism operates at any point in time is determined by the immediate history of prices. Those who are surprised by the persistence of stagflation have apparently misinterpreted the statistical evidence. The phenomenon has existed in many South American countries for years, only in the U.S. is it new and disturbing.

The results of Equation 5.8 suggest a price adjustment mechanism similar to that proposed in Chapter II. A slightly modified form of that structure will now be tested directly.

The price adjustment mechanism is defined by three difference equations in P and Y, see Equations Al through A3 of Appendix A. Note that as the learning factor elasticity E approaches one that the effect of lagged price and income expectations drop out of their respective equations, Al and A2. The expectational equations then become

1-

$$P_{t}^{\star} = P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)$$

$$Y_{t}^{\star} = Y_{t-1} \left(\frac{Y_{t-1}}{Y_{t-2}} \right)$$
(5.9)

With this modification the restriction that expected prices, P*, enter into the price equation, A3, with an exponential coefficient of one, the price equation to be tested becomes

$$\frac{P_{t}}{P_{t}^{\star}} = \left(\frac{Y_{t-1}}{Yf_{t-1}}\right)^{EF} \left(\frac{Y_{t-1}}{Y_{t-1}^{\star}}\right)^{EA}$$
(5.10)

This equation is log-linear and hence may be handled easily by least squares regression techniques. A proxy variable may be introduced as a measure of the ratio of actual income to normal full employment income. The variable used in the following tests was constructed from the labor unemployment rate according to the following rule:

$$z_t = 1 - \frac{\overline{v}_t}{100} + \frac{\overline{v}}{100}$$

where \overline{U} is the average rate of unemployment over the sample period. This variable of course captures only the effects of labor employment; it would be desirable perhaps to include some component to capture the employment rate effects of other factors, say the capacity utilization index. However the purpose of this section is primarily that of demonstrating the plausibility of the proposed price adjustment mechanism while at the same time showing that the mechanism is consistent with the notions generally held by economists about the way in which prices are formed. Use here of the unemployment rate alone will adequately serve both these ends.

The question of timing is central to an understanding of the workings of the proposed price mechanism. Economists today appear to be frustrated in their attempts to explain the simultaneous occurrence of both high rates of inflation and a high unemployment rate. This frustration may stem in part from an incomplete appreciation of the relative importance of the various factors which influence price changes and the timing of the effects of each of these factors -- the nature of the structure of lags in the effects produced by each of these factors through time.

The hypothesis under investigation is that three factors combine to determine the price level at any point in time: (1) price expectations, or more generally the recent history of the general level of prices; (2) the relationship between the actual level of real output and what may be called the normal full employment level; and (3) the relationship between actual and expected levels of income. Since the third factor is of minor theoretical importance, in subsequent discussions it will be largely ignored. We will be concerned here with the questions of the relative importance and timing of the effects of each of these factors.

Consider as a working hypothesis the following proposition:

Prices over the near term, say over a period of the next few months, are largely dependent on the immediate history of price movements -- factor cost increases will be routinely passed on with perhaps some lag, commodity demand is price inelastic in the short run, firms tend initially to adjust to changing economic conditions by varying employment and output not by changing price; over a longer horizon however the effects of other macro economic aggregates become manifest --it is only within the framework of this longer horizon that macro economic stabilization policy may be used to influence the mechanism through which aggregate prices are determined.

The most obvious way to test such a proposition would be to estimate a distributed lag function in the various variables by regression techniques. For example

$$P_{t} = e^{\alpha_{0}} \begin{pmatrix} K & \alpha_{i} \\ \Pi & P_{t-i} \end{pmatrix} \begin{pmatrix} K & \left[\frac{Y_{t-i}}{Yf_{t-i}} \right]^{\beta_{i}} \\ i=1 & \left[\frac{Y_{t-i}}{Yf_{t-i}} \right]^{\gamma_{i}} \end{pmatrix}$$

$$(5.11)$$

If it were discovered that the α_i 's tended to be large and significant for recent time lags while the β_i and γ_i weights were small but significant throughout the lag structure, the empirical evidence would generally support the thesis.

As an exercise for the energetic student this approach was pursued extensively and unsuccessfully. The statistical problem is twofold. Of all the macro economic time series, those which measure aggregate price levels are the most well behaved. For any of the common price series the correlation between P_t and P_{t-1} is extremely high. Any technique, then, which predicts future prices from past prices is extremely hard to improve upon. Similarly the lagged values of the full employment variable are highly correlated as are lags in the income expectation variable.

Hence any attempt to estimate 5.11 is doomed by the extreme multicollinearity of the matrix of independent variables. The usual solution to multicollinearity problems is that of using a priori information to construct weighted

averages of the collinear variables. Such a procedure is clearly inappropriate when the principal question at issue is in fact the nature of that weighting structure. In an attempt to attack this issue while avoiding the multicollinearity problems which arise in distributed lag estimation a more roundabout technique was employed.

In Equation 5.10 a relationship is proposed between prices in the current time period and other variables in preceding periods. No assumptions have been made however about the time duration of a particular period. Let j be the duration of one "period" in quarters. Equation 5.10 then becomes

$$\frac{P_{t}}{P_{t}^{*}} = \left(\frac{Y_{t-1j}}{Y_{f}_{t-1j}}\right)^{EF} \left(\frac{Y_{t-1j}}{Y_{t-1j}^{*}}\right)^{EA}$$
(5.12)

where

$$P_{t}^{\star} = P_{t-1j} \left(\frac{P_{t-1j}}{P_{t-2j}} \right)$$
$$Y_{t}^{\star} = Y_{t-1j} \left(\frac{Y_{t-1j}}{Y_{t-2j}} \right)$$

Estimation of 5.12 for j = 1, 2, 3, ... will provide some qualitative information about the timing relationships of the proposed price mechanism. A summary of such an exercise is contained in Table 5.1. The equations were first estimated by ordinary least squares and the first order residual autocorrelation component, $\hat{\rho}$, was obtained. The data was then

j		EO	EA	EF	ρ	dw ^a	đf ^b
1	lst pass	.0001 (.0004)	015 (.040)	.0077 (.038)	31*	2.64	89
	2nd pass	.0001 (.0003)	009 (.037)	.010 (.027)		2.13	
2	lst pass	.0007 (.0006)	052* (.029)	.051 (.060)	.43*	1.15	86
	2nd pass	.0008 (.001)	070* (.030)	(.008) (.087)		1.54	
3	lst pass	.0014* (.0009)	.011 (.028)	.148* (.087)	.76*	0.62	83
	2nd pass	.0014 (.0022)	026 (.028)	.204** (.143)		1.62	
4	lst pass	.0027* (.0011)	.071* (.028)	.296* (.111)	.818*	0.37	80
	2nd pass	.0043 (.0035)	.013 (.027)	.269** (.173)		1.55	

Table 5.1. Statistical estimates of Equation 5.12 parameters

^adw: Durbin Watson.

^bdf: Total degrees of freedom.

* Significant at 5%.

** Significant at 10%.

transformed and the system reestimated. An exponential scale factor of the form e^{EO} was appended to the RHS of 5.12.

These results are generally consistent with all important aspects of the working hypothesis proposed initially. It is well known that current prices may be predicted with remarkable accuracy by examining past price behavior. If one wishes to predict prices one or two quarters into the future no further information is helpful. For longer projections relevant additional information is provided by the current unemployment rate. The income expectations variable apparently provides no useful information. Macro economic stabilization policy may be expected to exert its influence on prices only over a relatively long period.

Note further that the impact of the level of employment upon prices is somewhat weak even where significant statistically. The estimated elasticity, EF, is 0.26 for the case where the time period under consideration is one year in duration. The variable clearly influences the process of price formation, but does not dominate it. Suppose, as a hypothetical example, that an extended period of rapidly rising aggregate demand coupled with a rapidly growing money stock combine to produce a high rate of inflation in an economy governed by a macro economic structure such as that proposed here. Suddenly government authorities become concerned about this inflation and react with the sudden imposition of a tight fiscal and monetary policy. The immediate impact will be on real output and employment. Gradually the employment decline will have an impact on the price level mechanism; however it may be several quarters

before the impact is noticeable and many years before adjustment is complete.

It is the very weakness of the employment-to-price feedback loop which allows short run Keynesian models to ignore it. Yet over the long term these weak feedbacks add up, giving rise to the standard tenet of money neutrality held by the Monetarists.

We now turn to the income equation of Chapter II. The proposed dynamic structure is recursive in nature. Prices at t are determined by the history of the system. Income is then determined jointly by exogenous demand, price, and the money stock. If the errors in the price equation are uncorrelated with errors in the income equation, ordinary least squares regression of income on prices and money should yield unbiased estimates of the coefficients.

Equation A4 was estimated as a log-linear function. This yielded an estimate of first order autocorrelation of .95 in the error terms. It was decided therefore to estimate A4 as a linear function of the percentage rates of change in the respective variables, a technique nearly equivalent to a .95 autocorrelation transformation with logarithmic variables. This form also emphasizes the "marginal", dynamic nature of the proposed relationship. All exogenous demand factors were ignored, their average effect is hopefully lumped into the intercept. To the extent that exogenous demand is correlated

with monetary or price movements these coefficients will be biased however.

Ordinary least squares estimation yielded

$$\frac{Y}{Y} t = \underbrace{0.86*}_{(0.23)} + \underbrace{0.41*}_{(0.21)} \left(\frac{P}{P} t \right) + \underbrace{0.27*}_{(0.09)} \left(\frac{M2}{M2} t \right)$$
(5.13)
d.w. = 1.14 $\hat{\rho} = .44*$ $S_e^2 = 1.02$ 91 degrees of freedom
The system was also estimated using the narrowly defined
money stock.

$$\frac{Y}{Y}t = 1.06^{*} + .49^{*}\left(\frac{P}{P}t\right) + .21^{*}\left(\frac{M1}{M1}t\right)$$
(5.14)
d.w. = 1.13 $\hat{\rho} = .44$ $S_{e}^{2} = 1.04$ 91 degrees of freedom

By transforming the data using the estimates for first order autocorrelation improved efficiency estimators were obtained.

$$\frac{Y}{Y}t = .92* + .44* \left(\frac{P}{P}t\right) + .23* \left(\frac{M2}{M2}t\right)$$
(5.15)

d.w. = 1.84 $S_e^2 = 0.83$ 91 degrees of freedom

$$\frac{Y}{Y}t = 1.18^{*} + .41^{*}\left(\frac{P}{P}t\right) + .14^{*}\left(\frac{M1}{M1}t\right)$$
(5.16)
d.w. = 1.87 $S_{e}^{2} = 0.84$ 91 degrees of freedom

As was mentioned previously, the legitimacy of this approach to recursive system estimation is dependent on the independence of the error terms in the price equation and the error terms of the income equation. Given the nature of the system in question it would be reasonable to expect some positive correlation however. To avoid this possibility, price estimates from the price regression, \hat{P}_t , were used to construct an instrumental price variable for use in the income regression.

$$\frac{\underline{Y}}{\underline{Y}} t = 1.30^{*} - 0.18 \left(\frac{\underline{\hat{P}}}{\underline{\hat{P}}} t\right) + 0.25 \left(\frac{\underline{M2}}{\underline{M2}} t\right)$$

d.w. = 1.29 $\hat{\rho}$ = .36 S_e^2 = .91 79 degrees of freedom The data was transformed for autocorrelation.

$$\frac{Y}{Y}$$
't = 1.24* + 0.186** $\left(\frac{\hat{P}}{P}$ 't $\right)$ + 0.20* $\left(\frac{M2}{M2}$ 't $\right)$

d.w. = 1.83 $S_e^2 = .78$

Considering the level of oversimplification inherent in the construction of this test, the essential qualitative relationship between income, prices, and money is consistent with that proposed in the theoretical model.

CHAPTER VI

Section A

The principal weak link in contemporary macroeconomic analysis is the failure to integrate the relationships which determine the level of real output with the processes which determine the level of aggregate prices. Until recently analysis has not addressed the question of how changes in nominal income are divided between changes in real output and prices. Much of macroeconomic analysis focuses on questions of extremely short horizon in which this division question may be safely ignored. The discussion and formulation of policies which make claims to intertemporal optimality cannot rely on such simplifications.

All elements necessary for the formation of an integrated theory are part of the standard body of knowledge of macro analysis. Building on this foundation a model was constructed in which both prices and real output appear as endogenous variables. This model was shown to be consistent with the principal long run properties of the monetarist position (money neutrality, a monetary theory of the price level, real output consistent with Walrasian full employment); further the model displays the usual short run Keynesian properties (monetary movements affect the level of real output and interest rates, prices are sticky in the short run). In addition under

certain conditions the model displays several interesting cyclic properties: procyclic behavior of inflation rates, interest rates, and velocity.

By making use of this model the problem of the formulation of monetary stabilization policy was treated as a topic in intertemporal dynamic optimization. Conceptually the problem was posed as a problem in optimal control theory. To avoid the computational difficulties of computing a general optimal feedback solution to the problem, an optimal monetary policy for several particular problems was located using an iterative dynamic programming algorithm. A cyclically varying exogenous demand factor was introduced into the model. It was shown that there exists an optimal control which results in normal full employment with no inflation throughout the cycle: this was termed the Golden Rule zero cost control path. Next control was allowed to deviate from this path during early phases of the cycle to allow analysis of the system with initial conditions not on the Golden Rule path. Optimal control with these initial conditions did not result in a zero cost path. However the optimal path ultimately converges toward the Golden Rule path, a turnpike property. Together these two exercises suggest the necessity for a symmetric cyclic damping control policy: it is necessary to persue a tight money policy during boom periods to allow policy to achieve both full employment and price stability objectives

during recessions. Finally the effect of persuing various interest rate objectives was considered. It was found that consideration of an interest rate stability objective in conjunction with full employment and price stability objectives may significantly decrease the possibility of achieving the latter two goals.

Section B

The particular model developed in Chapter II was kept as simple as possible by intention. No permanent income effects were considered in the consumption equation. The investment equation ignores the distinction between the real rate of interest and the nominal market rate, and allows for no acceleration effects. Wealth and inflation effects were omitted from the money demand function. The money supply was assumed exogenous. There is no linkage between past levels of investment and the level of full employment output. These clearly are oversimplifications which could be remedied easily in a more complete model.

Modifications of a more complex nature result from changes which extend the dimensionality of the model or introduce stochastic considerations. The present model is completely determined by some ten state variables plus two exogenous noncontrollable imputs and the exogenous control variable, the money stock. Extension of the price adjustment lags and

subdivision of consumption or investment demand into major components increases the number of state variables and hence the computational complexity of the system. The introduction of random elements requires reformulation of the objective function in expectational terms and treatment of stochastic differential equations.

The dynamic programming solution algorithm, while providing a cheap and simple method for computing solutions to particular problems, does not provide a solution for the optimal feedback control rule necessary for day to day policy formulation. By extending the analysis somewhat further it should be possible to discover a rule of the form: if the state of the is X_t at time t, and if estimates exist for exogenous demand and the full employment level of GNP, then the optimal level of the money stock is M_t^* . The present study demonstrates the gains to be made if optimal controls are employed. The feedback rule provides the necessary tools for specifying such a policy on a day to day basis.

BIBLIOGRAPHY

- Aris, R. 1964. Discrete dynamic programming. Blaisdell Publishing Company, Waltham, Mass.
- Athans, Michael, and Peter L. Falb. 1966. Optimal control: An introduction to the theory and its application. McGraw-Hill, New York.
- Bellman, R. E. 1957. Dynamic programming. Princeton University Press, Princeton, N.J.
- Bellman, R., and S. Dreyfus. 1962. Applied dynamic programming. Princeton University Press, Princeton, N.J.
- Bhatia, R. J. 1962. Profits and the rate of change in money earnings in the United States, 1935-1959. Economica 29: 255-262.
- Blackwell, D. 1962. Discrete dynamic programming. Annals of Math. Stat. 33:719-726.
- Brofenbrenner, Martin, and F. D. Holzman. 1963. Survey of inflation theory. Am. Econ. Rev. 53:593-661.
- Brown, A. J. 1955. The great inflation, 1939-1951. Cambridge University Press, London.
- Cagan, Phillip. 1956. The monetary dynamics of hyperinflation. Pages 25-117 in Milton Friedman, ed. Studies in the quantity theory of money. University of Chicago Press, Chicago, Ill.
- Cagan, Phillip. 1965. Determinants and effects of changes in the stock of money, 1875-1960. Columbia University Press, New York.
- Chow, G. C. 1970. Optimal stochastic control of linear economic systems. Journal of Money, Credit and Banking 2(3):291-302.
- Christ, Carl. 1971. Econometric models of the financial sector. Journal of Money, Credit and Banking 3(2,2): 419-449.
- Curry, H. B. 1944. The method of steepest descent for nonlinear minimization problems. Quarterly of Applied Math. 2:258-262.

- DeLeeuw, Frank, and Edward M. Gramlich. 1968. The Federal Reserve - MIT econometric model. Fed. Res. Bull. 54:11-40.
- Dicks-Mireaux, L. A., and J. C. R. Dow. 1959. The determinants of wage inflation: United Kingdom, 1946-1956. Jour. Royal Stat. Soc. 122(2):145-174.
- Dusenberry, James. 1950. Mechanics of inflation. Rev. Econ. Stat. 32:144-149.
- Dusenberry, James, Gary Fromm, and Lawrence Klein. 1965. The Brookings quarterly econometric model. Rand McNally, Chicago, Ill.
- Eckstein, Otto, and Gary Fromm. 1959. Steel and the postwar inflation. Joint Economic Committee Study Paper No. 2: 1-38.
- Economic Report of the President. 1974. U.S. Government Printing Office, Washington, D.C.
- Evans, Michael K., and Lawrence R. Klein. 1967. The Wharton econometric forecasting model. University of Pennsylvania, Wharton School, Philadelphia, Pa.
- Fair, Ray C. 1974. On the solution of optimal control
 problems on maximization problems. Annals of Econ. and
 Soc. Meas. 3(1):135-154.
- Felix, David. 1956. Profit inflation and industrial growth: The historic record and contemporary analogies. Quart. Jour. Econ. 70:441-463.
- Fisher, Irving. 1926. A statistical relation between unemployment and price changes. Int. Lab. Rev. 13(6):785-792.
- Ford, David I. 1971. The monetary theory of nine recent quarterly econometric models of the United States, comment. Journal of Money, Credit and Banking 3(2,2): 450-460.
- Friedman, Milton. 1956a. The quantity theory of money -- a restatement. Pages 3-21 in Milton Friedman, ed. Studies in the quantity theory of money. University of Chicago Press, Chicago, Ill.
- Friedman, Milton, ed. 1956b. Studies in the quantity theory of money. University of Chicago Press, Chicago, Ill.

- Friedman, Milton. 1957. A theory of the consumption function. Princeton University Press, Princeton, N.J.
- Friedman, Milton. 1959. The demand for money: Some theoretical and empirical results. Jour. of Pol. Econ. 67(4):327-351.
- Friedman, Milton. 1968. The role of monetary policy. Am. Econ. Rev. 58:1-17.
- Friedman, Milton. 1969. Interest rates and the demand for money. Pages 141-155 in Milton Friedman, ed. The optimum quantity of money and other essays. University of Chicago Press, Chicago, Ill.
- Friedman, Milton. 1970. A theoretical framework for monetary analysis. Jour. of Pol. Econ. 78(2):193-239.
- Friedman, Milton, and Anna Schwartz. 1970. A monetary history of the United States, 1867-1960. Princeton University Press, Princeton, N.J.
- Hansen, Bent. 1951. A study in the theory of inflation. Allen and Unwin, London.
- Holt, C. C. 1962. Linear decision rules for economic stabilization and growth. Quart. Jour. of Econ. 76:20-45.
- Holzman, Franklyn D. 1960. Inflation: Cost-push and demandpull. Am. Econ. Rev. 50:20-42.
- Johnston, J. 1972. Econometric Methods (2nd ed.). McGraw-Hill, New York.
- Keller, Elmo A., Jr. 1972. The computation of optimal growth in economic models. Unpublished Ph.D. dissertation. Iowa State University, Ames, Ia.
- Keynes, John Maynard. 1936. The general theory of employment, interest and money. Macmillan, New York.
- Lerner, Eugene M. 1956. Inflation in the Confederacy, 1861-65. Pages 163-175 in Milton Friedman, ed. Studies in the quantity theory of money. University of Chicago Press, Chicago, Ill.
- Lipsey, Richard. 1960. The relation between unemployment and the rate of money wage rates in the United Kingdom, 1862-1957: A further analysis. Economica 27:1-31.

- Machlup, Fritz. 1960. Another view of cost-push and demandpull inflation. Rev. Econ. Stat. 42:125-139.
- Patinkin, Don. 1959. The Israel economy: The first decade. Israel University Press, Jerusalem.
- Phelps, Edmund S. 1961. A test for the presence of cost inflation in the United States, 1955-57. Yale Econ. Essays 1:28-69.
- Phelps Brown, E. H. 1962. Wage drift. Economica 29:339-356.
- Phillips, A. W. 1958. The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1862-1957. Economica 25:283-299.
- Pierce, James L. 1969. Some simple rules for the conduct of monetary policy. Board of Gov. of Fed. Res. Spl. Stud. Paper 1.
- Pierce, James L. 1974. Quantitative analysis for decisions at the Federal Reserve. Annals of Econ. and Soc. Meas. 3(1):11-19.
- Pindyck, Robert S., and Steven M. Roberts. 1974. Optimal policies for monetary control. Annals of Econ. and Soc. Meas. 3(1):207-237.
- Pindyck, Robert S. 1972. Optimal stabilization policies via deterministic control. Annals of Econ. and Soc. Meas. 1(4):385-390.
- Pontryagin, L. S., and others. 1962. A mathematical theory of optimal processes. Interscience Publishers, New York.
- Poole, William. 1967. Optimal choice of monetary policy instruments in a simple stochastic macro model. Board of Gov. of the Fed. Res. Spl. Stud. Paper 2.
- Reynolds, Lloyd G. 1960. Wage-push and all that. Am. Econ. Rev., Proc. 1960:195-204.
- Routh, G. 1959. The relation between unemployment and the rate of change of money wage rates: A comment. Economica 26:299-316.
- Samuelson, P. A., and R. M. Solow. 1960. Analytical aspects of anti-inflationary policy. Am. Econ. Rev. 50:177-194.

- Schlesinger, James R. 1957. The role of the monetary environment in cost-inflation. So. Econ. Jour. 24:12-27.
- Selden, R. T. 1959. Cost-push versus demand-pull inflation, 1955-57. Jour. Pol. Econ. 67:1-20.
- Sengupta, J. K. 1970. Optimal stabilization policy with a quadratic criterion function. Rev. Econ. Stud. 37:127-145.
- Shell, K., ed. 1967. Essays on the theory of optimal economic growth. M.I.T. Press, Cambridge, Mass.
- Thorp, Willard, and Richard Quandt. 1959. The new inflation. McGraw-Hill, New York.
- Uzawa, H. 1964. Optimal growth in a two sector model or capital accumulation. Rev. Econ. Stud. 31:2-24.

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APPENDIX A

A Summary of the Dynamic Model

Dynamic structure:

$$P_{t}^{\star} = P_{t-1}^{\star} \cdot \left(\frac{P_{t-1}}{P_{t-2}}\right) \cdot \left(\frac{P_{t-1}}{P_{t-1}^{\star}}\right)^{EW}$$
(A1)

$$Y_{t}^{*} = Y_{t-1}^{*} \cdot \left(\frac{Y_{t-1}}{Y_{t-2}}\right) \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{*}}\right)^{EW}$$
(A2)

$$P_{t} = P_{t-1}^{*} \cdot \left(\frac{Y_{t-1}}{Yf_{t-1}}\right)^{EF} \cdot \left(\frac{Y_{t-1}}{Y_{t-1}^{*}}\right)^{EA}$$
(A3)

$$Y_{t} = k \cdot B_{t}^{\delta} \cdot P_{t}^{\delta} \cdot M_{t}^{\gamma}$$
(A4)

Static structure:

$$Y_{t} = C_{t} + I_{t}$$
 (A5)

$$C_{t} = b_{c} \cdot Y_{t}$$
 (A6)

$$I_{t} = B_{t} \cdot r_{t}^{EI}$$
 (A7)

$$M_{t}^{d} = M_{t}^{s}$$
(A8)

.

$$M_{t}^{d} \cdot b_{m} \cdot r^{EM} = Y_{t}$$
 (A9)

$$M_t^s = M_t$$
, exogenous (A10)

Full employment condition:

$$Yf_{t} = \frac{Yf_{t-1}}{P_{t-1}} \cdot (1 + RYf) \cdot P_{t}$$
(All)

Boundary conditions:

where:

$$k = b_{m} \cdot (1-b_{c})$$

$$B_{t} = b_{I} \cdot \prod_{i=1}^{t} \{(1 + RBI_{i})(1 + RYf)\}: \text{ exogenous demand}$$

$$\delta = (\frac{-EM}{EI - EM})$$

$$\gamma = (\frac{EI}{EI - EM})$$

$$\delta + \gamma = 1$$

$$\delta, \gamma > 0$$

Principal Coefficients

EI : Interest elasticity of investment EI < 0
EM : Interest elasticity of velocity; hence (-EM) is the interest elasticity of money demand
EW : Learning factor elasticities in the expectation functions
EA : Income Accelerator learning factor in the price function
EF : Employment Ratio impact elasticity in the price function

Parameter values used in simulation

BC=0.9D0 BI=45.D0 BM=1.5D0 EA=0.3D0 EF=0.5D0 EI=-0.4D0 EM=0.2D0 EW=0.1D0 M(1) = 100.000NPASS=0 P(1)=1.0D0PP(1)=1.0D0 PP(2) = 1.0D0PERP(1) = 0.0PERPP(1)=0.0PERY(1) = 0.0PERYP(1)=0.0RBI=0.0 RYF=0.0 YF(1)=216.338 YP(1) = 216.338YP(2) = YP(1) * (1.0D0 + RYF)

Variable Dictionary

- Y₊ : Gross National Product at nominal prices
- Ct : Consumption demand; or more generally all income dependent demand components
- I : Investment demand; or more generally all income independent demand components
- r_+ : The market rate of interest
- M^d₊ : Money demand
- M^S₊ : Money supply
- M₁ : Money stock; exogenous
- P₊ : Price level
- P^{*}₊ : Anticipated price level; Computer Code PP
- Y^{*}₊ : Anticipated GNP; Computer Code YY

Yt : Full Employment GNP at nominal prices
yt : Real GNP, Yt/Pt
yt : Anticipated Real GNP

APPENDIX B

Data Sources

The money stock

The monetary data used in this study were compiled from seasonally adjusted monthly data published regularly in the Federal Reserve Bulletin published by the Board of Governors of the Federal Reserve System. The various data time series were most recently revised and published in February 1973. Historical data back to 1952 was obtained from the revision; later data from subsequent issues. Quarterly series were constructed from simple averages of monthly data. The narrowly defined money stock, M1, consists of (1) currency outside of the Treasury, Federal Reserve Banks, and vaults of all commercial banks; (2) demand deposits at all commercial banks other than those due to domestic commercial banks and the U.S. government; (3) foreign demand balances at Federal Reserve Banks; less (4) cash items in the process of collection and Federal Reserve float. This corresponds to the M1 definition used by the Federal Reserve. The broad money stock definition, M2, employed consists of M1 plus all time deposits at commercial banks other than those due to domestic commercial banks and the U.S. government. This definition differs from that used by the Federal Reserve by the inclusion of large denomination negotiable Certificates of Deposit (over \$100,000) held by large weekly reporting banks.

These C.D.'s have grown rapidly in recent years -- from \$10 billion in 1969 to over \$45 billion in 1974. They now constitute a major source of funds for the commercial banking system.

Prices and output

The Gross National Product series compiled by the Department of Commerce was used as a measure of the total market value of final goods and services produced in a given time period. The series is estimated for quarterly periods, seasonally adjusted, and extrapolated to annual rates. A price-deflated series Gross National Product in Constant (1958) Dollars is also estimated by dividing broadly disaggregated components of GNP by their respective price indices. The implicit price deflator is simply the ratio of these two indices. The resulting ratio in principal takes the form of a

Pasche index, $\frac{\Sigma P_t w_t}{\Sigma P_0 w_t}$. Hence comparison of sequential values captures both the effects of price increases and changes in output composition. This is not a wholly undesirable characteristic however. Use of this index, furthermore, avoids some of the systematic bias found in both the Consumer Price Index and Wholesale Price Index.

The wage index

Two proxies for average wages were used in this study. The first is an index of average hourly wages of production line workers in manufacturing establishments compiled by the Department of Labor and published in the Survey of Current Business. This index includes overtime and shift premiums. It is hoped that this provides an index of per unit output labor cost. The second is an index of average weekly gross earnings in manufacturing establishments. This too includes all premiums. It is hoped that this provides an index of "take home pay", at least in the percentage rate of change form. This series was constructed by multiplying the average wage by the length of the average work week.

Monthly data was averaged to obtain a quarterly series; percentage rates of change were computed. This series was then regressed on quarterly dummy variables to obtain a seasonally adjusted series.

APPENDIX C

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Computer Program Listing

```
C
С
    CALLING PROGRAM FOR CHAPTER 4 SIMULATIONS
С
       REAL*8 TCOST (100, 3), SIG (100), COST (100), BELL (100)
       REAL*8 Y(100), YP(100), P(100), PP(100), R(100), V(100)
       REAL*8 M (100), YF (100)
       REAL*8 PERY(100), PERP(100), PERYP(100), PERPP(100)
       REAL*8 BC, BI, BM, EI, EM, EA, EV, EF, RM, RBI, RYF
       REAL*8 BB(100)
       REAL*8 ZETA, STPP
       COMMON YF,Y,YP,P,PP,R,V,M,PERY,PERP,PERYP,PERPP
       COMMON BC, BI, BM, EI, EM, EA, EW, EF, RM, RBI, RYF, NPASS, NN
       COMMON /AREA1/ TCOST, SIG, COST, BELL, ZETA, STPP
       COMMON /AREA2/ BB
С
С
     INITIALIZE VARIABLES
       NN=20
       BC=0.9D0
       BI=45.D0
       BM=1.5D0
       EA=0.3D0
       EF=0.5D0
       EI = -0.4D0
       EM=0.2D0
       EW=0.1D0
       M(1) = 100.000
       NPASS=0
       P(1) = 1.0D0
       PP(1) = 1.0D0
       PP(2) = 1.0D0
       PP(3) = PP(2)
       PERP(1) = 0.0
       PERPP(1) = 0.0
       PERY(1) = 0.0
       PERYP(1) = 0.0
       RBI=0.0
       RYF=0.0
       STPP=0.01D0
       YF(1) = 216.338
       YP(1) = 216.338
       YP(2) = YP(1) * (1.0D0 + RYF)
       YP(3) = YP(2) \neq (1.0D0 + RYP)
       ZETA=1.DO
С
С
     COMPUTE TRIAL CONTROL FOR M
```

С

С	· ·					
	CALL MGEN					
С						
C. C	COMPUTE EXOGENOUS DEMAND CYCLE					
~	CALL HOGS					
C	COMPUTE STATE TRAJECTORY					
C ~	CALL Y34Y					
	PRINT OUTPUT					
c	CALL PRNTR					
č	COMPUTE COST FUNCTION					
с ~	CALL OBJ					
c c	ITERATE THE DYNAMIC PROGRAMMING SOLUTION ROUTINE					
-	700 DO 800 JP=1.5					
	800 CALL OUAM					
	CALL PRNTR					
	IF (NPASS.LT.10) GOTO 700					
С						
c c	REDUCE STEPSIZE FOR CLOSER CONVERGENCE					
	STPP=0.005D0					
	IF (NPASS.LT.15) GOTO 700					
	900 CONTINUE					
	999 STOP					
	END					

.

-

```
SUBROUTINE Y34Y
C
С
    COMPUTE TIME PATH FOR COMPLETE DYNAMIC SYSTEM
С
С
    REQUIRES MONEY STOCK IMPUT AND PARAMETER INITIALIZATION
С
      REAL*8 Y(100), YP(100), P(100), PP(100), R(100), V(100)
      REAL*8 M (100), YF (100)
      REAL*8 PERY(100), PERP(100), PERYP(100), PERPP(100)
      REAL*8 BC, BI, BM, EI, EM, EA, EW, EF, RM, RBI, RYF
      REAL*8 BB(100)
      COMMON YF, Y, YP, P, PP, R, V, M, PERY, PERP, PERYP, PERPP
      COMMON BC, BI, BM, EI, EM, EA, EW, EF, RM, RBI, RYF, NPASS, NN
      COMMON /AREA2/ BB
      DO 20 II=1 NN
      IP(II.LE.2)GOTO 10
С
С
   COMPUTE EXPECTED PRICE AND INCOME
С
   EXPECTATIONS ARE BASED ON PAST EXPECTATIONS
С
    RATE OF INCREASE AND CORRECTED BY LEARNING FACTOR
С
С
       PP(II) = PP(II-1) **(1.0D0-EW) *P(II-1) **EW*P(II-1) /P(II-2)
       YP(II) = TP(II-1) ** (1.0D0-EW) *Y(II-1) ** EW*Y(II-1) /Y(II-2)
   10 CONTINUE
      IF (II.EQ. 1) GOTO 11
С
С
    COMPUTE PRICE LEVEL
С
С
    PRICES ARE A FUNCTION OF EXPECTED LEVELS
С
    CORRECTED BY A LEARNING FACTOR
С
     AND A % FULL EMPLOYMENT FACTOR
С
С
    YF IS FULL EMPLOYMENT LEVEL OF INCOME IN CURRENT PRICES
С
       P(II) = PP(II) * (Y(II-1) / YP(II-1)) **EF*(Y(II-1) / YP(II-1))
       YF(II) = YF(II-1) / P(II-1) * P(II) * (1. D0 + RYF)
    11 CONTINUE
С
     COMPUTE NOMINAL INCOME
С
С
       Y(II) = (M(II) + BH) + (EI/(EI-EM)) + ((1.DO-BC)/(P(II) + BB(II)))
      1** (EE/(EI-EM))
C
С
     COMPUTE COMPONENTS OF AGGREGATE DEMAND, R, V, ETC.
С
```
```
V(II) =Y(II) /M(II)
R(II) = (Y(II) * (1.0D0-BC) / (P(II) *BB(II))) ** (1.0D0/EI)
IF(II.EQ.1) GOTO 20
PERY(II) = (Y(II) -Y(II-1)) /Y(II-1) *100.D0
PERP(II) = (P(II) - P(II-1)) /P(II-1) *100.D0
PERYP(II) = (YP(II) - YP(II-1)) /YP(II-1) *100.D0
PERPP(II) = (PP(II) - PP(II-1)) / PP(II-1) *100.D0
20 CONTINUE
NPASS=NPASS+1
RETURN
END
```

٠

```
SUBROUTINE OUAM
Ċ
С
C
    A MODIFIED GRADIENT DYNAMIC PROGRAMMING ALGORITHM
    TO COMPUTE OPTIMAL CONTROLS FOR THE DYNAMIC SYSTEM
С
С
    OF CHAPTER 2 FOR A PARTICULAR COST FUNCTION
С
      REAL*8 TCOST (100, 3), SIG (100), COST (100), BELL (100)
      REAL*8 Y(100), YP(100), P(100), PP(100), R(100), V(100)
      REAL*8 M (100), YF (100)
      REAL*8 PERY (100), PERP (100), PERYP (100), PERPP (100)
      REAL*8 BC, BI, BM, EI, EM, EA, EW, EF, RM, RBI, RYF
      REAL*8 BB(100)
       REAL*8 D,Z1,Z2
       REAL*8 ZETA, STPP
      COMMON YF,Y,YP,P,PP,R,V,M,PERY,PERP,PERYP,PERPP
       COMMON BC, BI, BM, EI, EM, EA, EW, EF, RM, RBI, RYF, NPASS, NN
      COMMON /AREA1/ TCOST, SIG, COST, BELL, ZETA, STPP
       COMMON /AREA2/ BB
       DO 3 J=1,NN
    3 SIG(J) = 0.0D0
       NDO=NN-2
    12 DO 30 J=1, 15
       II = NN - J + 1
       NK = 2
       D=1.005D0
С
С
    COMPUTE TCOST FUNCTIONS
С
    15 M(II) = M(II) *D
       Y(II) = (M(II) * BM) ** (EI/(EI-EM)) *((1.DO-BC)/(P(II) * BB(II))
      1**(EM/(EI-EM))
       DO 20 K=1,2
       II=II+1
       IF (II.LE.2) GOTO 10
       IF (II.GT.NN) GOTO 20
Ç
С
   COMPUTE EXPECTED PRICE AND INCOME
С
       PP(II) = PP(II-1) ** (1.0D0-EW) *P(II-1) **EW*P(II-1) /P(II-2)
       YP(II) = YP(II-1) **(1.0D0-EW) *Y(II-1) **EW*Y(II-1)/Y(II-2)
    10 CONTINUE
       IF (II.EQ. 1) GOTO 11
С
     COMPUTE PRICE LEVEL
       P(II) = PP(II) * (Y(II-1) / YF(II-1)) **EF*(Y(II-1) / YP(II-1))
       YF(II) = YF(II-1) / P(II-1) * P(II) * (1. D0 + RYF)
```

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```
11 CONTINUE
С
С
    COMPUTE NOMINAL INCOME
С
      Y(II) = (M(II) *BM) ** (EI/(EI-EM)) *((1.D0-BC)/(P(II)*BB(II))
     1** (EM/(EI-EM))
   20 CONTINUE
      II=II-2
      Z1=0.0
      22=0.0D0
       DO 21 JL=1,3
      IF((II+JL-1).GT.NN) GOTO 21
      Z1=(100.D0*(Y(II+JL-1)-YF(II+JL-1))/YF(II'JL-1))**2+Z1
       IF((II+JL-2).LT.1) GOTO 21
      Z2 = (100.D0*(P(II+JL-1)-P(II+JL-2))/P(II+JL-2))**2+22
   21 CONTINUE
       TCOST(II, NK) = Z1 + Z2 \neq ZETA
       M(II) = M(II) / D
       IF (NK.EQ.3) GOTO 22
       IF (NK.EQ.1) GOTO 211
       NR = 1
       D=1.D0
       GOTO 15
  211 D=0.995D0
       NK=3
       GOTO 15
C
C
    FIND SMALLEST TCOST
С
   22 IF (TCOST (II, 2).LT.TCOST (II, 3)) GOTO 23
       IF (TCOST(II, 3) .GT.TCOST(II, 1)) GOTO 24
       SIG (II) = -1.00
       GOTO 25
    23 IF (TCOST (II, 2) .GT. TCOST (II, 1)) GOTO 24
       SIG(II) = 1.00
       GOTO 25
    24 SIG (II) =0.0
    25 CONTINUE
       IF (SIG (II)) 26,30,27
    26 CONTINUE
    27 CONTINUE
С
С
     MODIFY M(II) AS APPROPRIATE
С
    29 H(II) = H(II) * (1.D0+SIG(II) *STPP)
    30 CONTINUE
С
С
     NO ATTEMPT IS MADE TO COMPUTE BEST STEPSIZE
С
     ROUTINE MAY NOT CONVERGE IN NEIGHBORHOOD OF OPTIMAL PATH
C
     HENCE A SMALL NUMBER OF ITERATIONS IS RECOMMENDED
```

STEPSIZE, STPP, SHOULD BE REDUCED IN LATER ITERATIONS С С WRITE (6,56) 56 FORMAT ('0',' TCOST 1 TCOST 2 1'TCOST 3 SIG SUBROUTI NE WRITE (6,57) ((TCOST(J,K),K=1,3),SIG(J),J=1,NDO) 57 FORMAT (* *,4F20.5) С USING MODIFIED N., COMPUTE SYSTEM TRAJECTORY С С CALL Y34Y С COMPUTE VALUE OF COST FUNCTION С С CALL OBJ С ALL ITERATIONS MUST BE CALLED FROM MAIN PROGRAM C С RETURN END **SENTRY**